Choosing optimal reverse channel structures for the collection of used products

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Abstract

The re-manufacturing of used products has become more important in literature and practice. Governmental legislation forces manufacturers to take care of their end-of-life products. Additionally, re-manufacturing may increase a companies' revenue through direct savings in production costs by the recovery of valuable material. With these external and internal developments, there is a growing interest of manufacturers in determining an optimal channel for the collection of used products. The overall objective of this optimisation lies in the maximisation of the companies' profit. Therefore, the problem of increasing waste streams of end-of-life products need to be addressed by identifying the most profitable reverse channel structure to collect and re-manufacture used products.

Three different collection channel options are modelled as decentralised decision-making systems. Therefore, a game theory approach is applied. The first channel is the manufacturer carrying out the collection. The retailer making use of the retail store network to collect from customers and sell back to the manufacturer describes the second channel. A third-party logistics service provider acts as a third channel for collecting and selling returns. The thesis focuses on the detailed cost of collection that each potential collecting agent accommodates. A non-cooperative game between the three collecting agents is modelled first, followed by the extension to a cooperative game. The cooperation can be caused by external influences like legislative regulations or by a change in perspective. The stability of both versions of the game is evaluated by changing single parameters. Additionally, by changing the market scenario, the influence of the market environment on the channel choice is investigated in particular.

The benchmark scenario of the non-cooperative and cooperative version of the game is stable in its parameters. In general, changes in single parameters influence the level of the highest payoff achievable by each player. In the non-cooperative version of the game the manufacturer gains the highest payoffs followed by the retailer, as both benefit from the sales of new products. If the market scenario is changed, this ranking only shifts with a change in the market area size. Therefore, the retailer obtains a profit higher than the manufacturer. The third-party is able to work with different clients, turning the collection of returns into a successful business. The results of the cooperative version of the game are consistent with the observations in the non-cooperative game. Forming the grand coalition is the best option to obtain the highest payoffs if collection rate fees are imposed externally. With a change of perception to the manufacturer, the same customer density identifies the retailer as an optimal collection channel. However, subcontracting the third-party obtains the highest payoff in the benchmark scenario as well as in the larger market areas. In conclusion, a cooperation between different options should be taken into account while designing optimal reverse channel structures for every scenario. Additionally, the point of view is crucial in choosing the partner to obtain the highest payoff.

Opsomming

Die hervervaardiging van gebruikte goedere word al hoe noodsaakliker in sowel die literatuur as in die praktyk. Regeringswetgewing dwing vervaardigers om produkte te verwerk wat die einde van hul raklewe bereik het. Die hervervaardiging van produkte kan tot voordeel van 'n maatskappy se inkomste wees deur direkte besparing in vervaardigingskoste wanneer waardevolle materiale herwin kan word. Met hierdie eksterne en interne ontwikkeling is daar 'n toenemende belangstelling van vervaardigers om 'n optimale kanaal vir die versameling van gebruikte produkte te skep. Die algemene doelwit van hierdie optimering lê in die maksimering van die maatskappy se wins. Daarom is dit noodsaaklik dat die probleem van toenemende afvalhope van produkte aan die einde van hul raklewe aangespreek word deur die mees winsgewende tru-kanaalstruktuur te skep waardeur hierdie produkte versamel en hervervaardig kan word.

Daar is drie verskillende versamelingskanale wat as modelle kan dien vir gedesentraliseerde besluitnemingsisteme. 'n Model wat op 'n speleteorie benadering gebaseer is, word gebruik. Die eerste kanaal is die vervaardiger wat die versameling behartig. Die tweede kanaal is die kleinhandelaar wat gebruik maak van die winkelnetwerk om die produkte van die kliënte te versamel en terug te verkoop aan die vervaardiger. Die derde kanaal is 'n onbetrokke logistieke diensverskaffer wat die produkte wat teruggegee is, versamel en herverkoop. Hierdie tesis fokus op die gedetailleerde koste van die versameling van elke potensiële versamelingsagent. 'n Niesamewerkingspel tussen die drie versamelingsagente is die eerste model, gevolg deur 'n uitbreiding na 'n samewerkingspel. Die samewerking kan veroorsaak word deur eksterne invloede soos wetgewende bepalings of deur 'n verandering in perspektief. Die stabiliteit van beide weergawes vanuit 'n speleteorie benadering word getoets deur enkele parameters te verander. Deur die mark scenario telkens te verander, word die invloed van die markomgewing op die keuse van die tipe kanaal ook ondersoek.

Die scenario wat die maatstaf vorm vir die nie-samewerkings en die samewerkings weergawe van die spel is stabiel in terme van die invoer parameters. Oor die algemeen word die vlak van die hoogste wins wat elke speler kan bereik, deur veranderinge in enkele van die parameters beïnvloed. In die nie-samewerking weergawe van die spel word die meeste wins deur die vervaardiger gemaak, gevolg deur die kleinhandelaar, aangesien albei voordeel trek deur die verkope van nuwe produkte. As die mark-scenario verander, verander die rangorde slegs met 'n verandering in die grootte van die markgebied, en word die verkoper se wins meer as die van die vervaardiger. Die derde party kan met verskillende kliënte werk en die versameling van goedere in 'n suksesvolle besigheid verander. Die uitslag van die samewerkings weergawe van die spel is konsekwent met dié van die nie-samewerkings weergawe. Die omvattende koalisie is die beste keuse om die hoogste wins te maak, indien die koste van die versameling ekstern gehef word.

Ter samevatting, 'n samewerking tussen verskillende moontlikhede moet in berekening gebring word wanneer die optimale tru-kanaal struktuur vir elke scenario geskep word. Voorts is hierdie standpunt van uiterste belang wanneer 'n vennoot gekies word om die hoogste wins te verseker.

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Any opinions or findings in this thesis are those of the author and do not necessarily reflect the view of Stellenbosch University.

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Glossary

Closed-loop supply chain can be defined as "the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time" [42, p. 10].

Diseconomies of scale factors rise the average costs in the long-run with an increase in the scale of production. Even though the unit cost may fall with an increase in output due to economies of scale, there are reasons that reverse this process eventually. There is a growth of bureaucracy with a growth of production to manage the difficulties in coordination and administration. If the market area of a company becomes too large, the transport cost may offset a large companies' scale economies of production. Even transporting full-truck-loads do not minimize the cost of transport per unit to an acceptable level if the customer is too far from the facility. Besides these internal factors, there are also external factors that can lead to diseconomies of scale such as traffic congestion in the geographic region of the production plant [4].

Economies of scale are factors that cause the cost of producing a unit to decrease as the output of units increases. There are internal and external economies of scale. Generally factors within the company ensure an optimal production size that is large. Especially with high fixed cost of operating a facility it is profitable to produce many units to distribute the cost over the units. Additionally, larger companies can invest in research and development and specialise in machinery and labour to increase the overall productivity. There are also non-technological factors that can lead to economies of scale such as discounts for buying in bulk from the supplier amongst others. Especially the cost of transport per unit can be cut down if the fixed cost of operating a vehicle is distributed amongst as many units as possibly fit in the truck. External economies arise due to the development of an industry. This development can lead to services that benefit all firms. Economies of scope on the other hand do not originate in a higher output but in the production of a range of related units [5, 6].

Game is any social situation involving two or more individuals [56]. It is described by the totality of the pre-defined rules of the game. A sequence of moves by the individuals make up a game. [91].

Move is the event of a choice between various alternatives made by either one of the players of a game or by some device subject to chance. The choice is made under the conditions of the pre-described rules of the game. All moves are the element components of a game [91].

Choice describes the specific alternative that is chosen in a concrete play. A sequence of choices make up one play whereas a game consists of a sequence of moves. The element components of a play are the choices [91].

Payoff is received by each player after each play. A negative number can be interpreted as a loss while a positive as a win. Payoffs do not necessarily need to be monetary, but can also be of a specific utility value [56].

Play describes every instance at which a game is played. The game is played, in a particular way, from the beginning to the end. A sequence of choices make up a play [91].

Player is every individual involved in a game. The two basic assumptions are that players act rational and intelligent [56].

Reverse logistics is "the process of planning, implementing and controlling backward flows of raw materials, in process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal" [15, p. 5].

Rules are absolute commands. If the pre-defined rules are violated, then the game ceases to be the game defined by the rules [91].

Stackelberg duopoly model is a model with one company that has most market power acting as the price leader and another company as the price follower. Therefore, it is a sequential game where the Nash equilibrium is identified by backward induction [71].

Strategy is freely chosen by each player. Only general principles influence the choice of strategy. Therefore, it is within each player's responsibility to use or reject strategies [91].

List of Reserved Symbols

Symbol	Meaning
Parameters	
α_ℓ	A parameter used to fit real data for the correctional factor with distance.
α_q	A parameter used to fit real data for the correctional factor with quantities.
b	A parameter for the average speed on the back-haul tour.
β	A positive parameter of the demand function.
c	The average unit cost of manufacturing.
${}^iC_A$	The acquisition cost per unit in player i's channel.
c_m	The cost of manufacturing a new product.
c_r	The cost of recovering a used product.
d	The average tour distance.
d_L	The average local tour distance per customer.
D	The demand.
Δ	The unit cost saving from recovery.
η_ℓ	A parameter used to fit real data for the correctional factor with distance.
η_q	A parameter used to fit real data for the correctional factor with quantities.
f	The annualized fixed cost of operating a service facility per unit.
F_i	The annualized fixed cost of operating a service facility of player i.
H_D	The direct shipping holding cost per unit.
H_M	The milk-run holding cost per unit.
I	The return on investment in collection activities.
k	A constant to describe the average tour distance.
K_L	The loading cost.
K_U	The unloading cost.
κ_i	The capacity limit player i's facility.
l	A parameter for the average speed on the local collection tour.
λ	The constant of the golden ratio search.
M	The milk-run transport cost per unit.
M_A	The milk-run transport cost integrated over the market area per unit.
M_C	The milk-run transport cost in continuous approximation per unit.
M_R	The vehicle routing cost per unit.
O	The customers.
p	The retail price per unit.
P_i	The payoff per unit of player i.
ϕ	A positive parameter of the demand function.
φ_i	The density constant of player i.

Ψ_ℓ	The correctional factor for economies of scale with distance.
Ψ_q	The correctional factor for economies of scale with quantities.
q	The quantity of a product.
q_{max}	The maximum truck capacity for a product.
r	The holding cost rate.
R_i	The scaling parameter for the investment cost of player i.
ρ_i	The customer or point density of player i.
S	The lot size.
S_{max}	The accumulation capacity of a collection point.
t	The transport cost with high truck capacity per distance per unit.
T	The transport cost per distance per unit.
T_D	The direct shipping transport cost per unit.
T_C	The direct shipping transport cost in continuous approximation per unit.
U	The transit time.
U_L	The transit time of local transport.
v	The vehicle capacity of a high volume truck.
V	The vehicle capacity.
V_{max}	A full truck load.
w	The wholesale price per unit.
z	A parameter describing non-desirable variations of the cost-ratio.

Sets	
$\mathcal{P}$	The set of players in the grand coalition.
$\emptyset$	The set of players in the empty coalition.

Variables	
$2w$	The width of a pick-up zone.
A_i	The service area of player i.
b_i	The transfer price per unit for player i.
C	The total cost of collection.
${}^iC_N$	The non-linear reverse logistics cost per unit in player i's channel.
${}^iC_D$	The discontinuous reverse logistics cost per unit in player i's channel.
ℓ	The distance between customer and service facility.
L	The length of a pick-up zone.
${}^i\Pi$	The payoff per unit in player i's channel.
u_i	The units attainable to player i.
Θ_i	The collection or return rate of player i.

List of Acronyms

EOQ: Economic order quantity

EU: European Union

CLSC: Closed-loop supply chain

MILP: Mixed integer linear programming

OEM: Original equipment manufacturer

RFID: Radio-frequency identification

RL: Reverse logistics

USA: United States of America

WEEE: Waste electrical and electronic equipment

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CHAPTER 1

Introduction

Contents

The establishment of trade transfers the ownership of goods or services in exchange for some form of currency. A network in which this exchange can be executed is constituted by a market. The demand created by a market can only be satisfied if requested items can be supplied. Therefore, some form of manufacturing need to transform raw materials into products. The transformation can only be carried out, if the raw materials are available at the plant, while the market can only be supplied if the products are shipped to the potential customers. The supply chain describes the steps of transport from one point of demand to the next, beginning with the supplier providing the raw material and ending at the customer in the market area. Throughout the years not only manufacturing processes have been optimised, but also supply chains became more and more efficient. With the rising efficiency, the supply of the markets increase. Additionally, items become more affordable to a larger section of the world-wide growing population that is creating a higher demand. However, the high rates of product supply creates a new problem: The increasing level of consumption results in an enormous waste stream of end-of-life-products.

This problem is addressed by replacing the one-way perception of a company turning raw material into goods that are sold to customers by the more holistic view of product life cycles. Therefore, the entire process from manufacturing to recovery is taken into consideration. Besides, legislation as well as customers expect companies to set the environmental impact of their products to a minimum. Especially the sustainable use of finite resources and the limited availability of disposal capacity together with the ecological impacts of waste trigger this change in perception.

1.1 Forward and reverse logistics

The change in perception goes hand in hand with a change in companies' processes. Not only facilities to manufacture but also to re-manufacture used products have to be included in the design of a plant. Besides, the design of the product itself has to allow the possibilities of recovery. In this context, traditional forward logistics are extended by the introduction of reverse logistics (RL). While forward logistics define the process flows from raw material that is being transformed into products and shipped to customers, reverse logistics describe the flows of material and information of used products from the customer back to the recovery facilities of the company or an external service provider. Forward combined with reverse flows result in the so-called closed-loop supply chain (CLSC). Therefore, the CLSC is responsible for all process flows in the life cycle of a product that is traded in a market.

1.2 Motivation

The motivation behind re-manufacturing used products and thus introducing reverse logistics to the traditional transport and manufacturing processes are two-fold. On the one hand, incorporating reverse logistics and product recovery into the manufacturing process can lead to financial benefits for a company. On the other hand, in certain countries the legislative imposes regulations that companies have to follow to reduce the waste stream created.

1.2.1 Economic reasons

The annual costs of commercial returns are, as estimated by Stock et al. [81], in excess of 100 billion US dollar. Two other examples, describing the experiences of Guide et al. [42], show that more than 700 million US dollar of functional recovered products of a computer network manufacturer were destroyed without taking advantage of the possibilities of recycling. Furthermore, the return cost of Hewlett-Packard mounts up to 2% of the total outbound sales with less than half of the values of these products being recovered. These examples show that most of the economic potential of recovery of used products and closed-loop supply chains is currently dumped on landfill sites all over the globe.

There are exceptions. Xerox, for example, turns end-of-life electronic equipment into new products containing recycled material. Therefore, Xerox implemented a programme that enables the re-use of complete end-products, the re-manufacturing or conversion into updated products, the re-use of major modules or subcomponents of products as well as the recycling of material. According to Xerox, the re-use of material, especially with toner cartridges, saves several million US dollar of raw material cost every year [94]. Moreover, Canon as well as Hewlett Packard undertake similar re-manufacturing activities to benefit from the economic potential of re-use.

The total volume of goods in the reverse flows of IBM sums up to 10 000 metric tons worldwide. IBM decided to offer different re-use options to recover the maximum of this value. With the help of the product recovery strategies that had been introduced, a financial benefit of several hundred US dollar as well as a reduction of land-filling and incineration to less than 4% are achieved by IBM's activities [28].

In general, re-manufacturing is more efficient on energy than other forms of recycling. The amount of energy needed can be reduced drastically, since products do not have to be broken up or processed chemically. More importantly, re-manufacturing is, according to Ginsburg [35], between 40% and 65% cheaper than manufacturing new products as material cost can be saved on a large scale. All these numbers only give a glimpse at the enormous economic potential of re-manufacturing activities incorporating CLSCs.

1.2.2 Legislative regulations

In the US, the total quantity of municipal solid waste grew from 88 million tons in 1960 to 256 million tons in 2006, according to the United States Environmental Protection Agency [89]. In 1960, 94% of the waste was landfilled or disposed, while only 6% was recovered. However, in the last decades the recovery and composting efforts have increased due to several regulations imposed on the manufacturers.

In Europe, the waste of electrical and electronic equipment was 9 million tonnes in 2005 and is expected to be 12 million tonnes by 2020. Therefore, it is one of the fastest growing waste streams in Europe [22]. The Waste Electrical and Electronic Equipment directive (WEEE) became law in Europe in 2003 to address this problem. The directive contains mandatory requirements with regards to the collection of end-of-life products to increase re-use and recycling. Similar legislations have been introduced in Canada, Japan and China [38].

The US and Europe seems to engage the strongest in environmental legislations. Therefore, manufacturers having their facilities located in one of those continents need to pose the question on how to deal with their end-of-life products in an early stage of the product development.

1.3 Problem description and thesis scope

Choosing the reverse channel structure that is optimal arises from the core of operations research with its overall aim of optimisation. With the application of operations research to the topic of re-manufacturing and reverse logistics, scientific methods are applied to provide a comprehensive decision support with a quantitative basis and objective perspective. Therefore, decisions controlling the operations of a system, such as choosing a particular structure for reverse logistics activities, is the subject matter of this project. The overall objective of the optimisation lies in the maximisation of a company's profit. A model that describes the system in a realistic way serving as a benchmark scenario will be modelled. Applying game theory, the variables that can be manipulated to maximise the objective are identified. Constraints set by market mechanisms build a framework for the investigation of this project.

The optimality of the reverse channel structure is influenced by the two-fold motivation. Economic reasons as well as legislative regulations result in a financial impact for the manufacturing company. While the economic reasons bring direct savings in material cost, the monetary aspect of the legislative regulations lies in avoiding fines that need to be paid if predefined collection rates are not reached. Therefore, the optimality of the reverse channel structure will be evaluated by the monetary effects of the implementation.

The effect of logistics on the choice of the optimal reverse channel structure will be evaluated. By modelling different transport network alternatives through different players, the options will be tested. Versions were these players cooperate or not, as well as various environments are investigated. The purpose of this thesis is to highlight the importance of reverse logistics by emphasising monetary benefits of re-manufacturing. On a theoretical basis, it will be shown that it is vital for manufacturers to engage into the design of reverse channel structures.

1.4 Thesis objectives

Towards the aim of choosing an optimal reverse channel, the following objectives will be pursued throughout the thesis.

OBJECTIVE I: Perform a literature review on the current body of scientific knowledge.

OBJECTIVE II: Model different reverse logistic options with a game theory approach.

OBJECTIVE III: Test the stability of the modelled game.

OBJECTIVE IV: Investigate different scenarios and versions of the modelled game.

OBJECTIVE V: Interpret the results.

OBJECTIVE VI: Draw conclusions on the choice of an optimal reverse logistics structure.

1.5 Thesis outline

Many real-world problems demand mathematical descriptions and solution approaches. There are several stages a problem needs to pass through to successfully apply mathematical theory. As a first step after the introduction of the topic in this chapter, the problem is identified in Chapter 2. An analysis of the current state of scientific knowledge characterises the problem and reveals gaps for further investigations. In Chapter 3 the problem is formulated as a mathematical model. In this second step, the problem is thus translated into mathematical terms. There are opposing elements that need to be balanced out in the formulation. On the one hand, to successfully analyse the problem the model has to work with simplifying assumptions to overlook minor details. On the other hand, to apply the conclusions of the study to the original problem, the model has to reflect the real-world situation adequately. The third step carried out in Chapter 4 is finding a solution to the problem. The solution of the problem is computed in various environments to reveal information about the different elements that can be influenced within the mathematical model. Therefore, the stability of the model as well as different market scenarios are tested. In Chapter 5 and Chapter 6 the results are translated back into the original context as the final step. Extensions help to create a more realistic picture of the problem as the two-fold motivation is taken into account by applying different perspectives. Conclusions are drawn in the final chapter. Additionally, a possible outlook on the future is given in Chapter 7 [85].

CHAPTER 2

The current state of scientific knowledge

Contents

The following sections will describe the body of scientific knowledge of the two topics that will be combined in this project: Reverse logistics and game theory. Game theory approaches are applied to the field of reverse logistics to change the point of view from a centralised planer that carries out all tasks arising from a closed-loop supply chain to a decentralised decision-maker [73]. This incorporation emphasises the characteristics of reverse logistics.

First of all, the topic of closed-loop supply chains and reverse logistics will be described in detail, followed by an overview over the tools of game theory. The combination of the two topics in selected articles serves as a basis for the description of the problem this thesis is going to address.

2.1 Closed-loop supply chains and reverse logistics

Background information on closed-loop supply chains and reverse logistics are described to illustrate the current body of knowledge available in literature. Future research opportunities will also be outlined to serve as a foundation for the research of this thesis.

2.1.1 Attempts to close the loop

Taking back used products from customers and re-using the entire product, modules, parts and components to recover the value added is the focus of closed-loop supply chains. Therefore, CLSCs include not only activities from traditional forwards supply chains but also activities from reverse supply chains. The additional activities are the acquisition of products from the customers, the shipment of returns to the service facilities, the determination of the return's quality condition, the re-manufacturing and the re-marketing [40].

All these process steps are displayed in Figure 2.1. Raw-materials are manufactured to final products at the plant. Afterwards, the products are either directly or indirectly distributed via a retailer to the customers. The forward supply chain incorporates these tasks. The reverse supply chain collects the used products from the customers and ships them to the recovery facility. The recovery facility, that can be included in the plant, supplies the manufacturing with recovered raw materials and disposes waste.

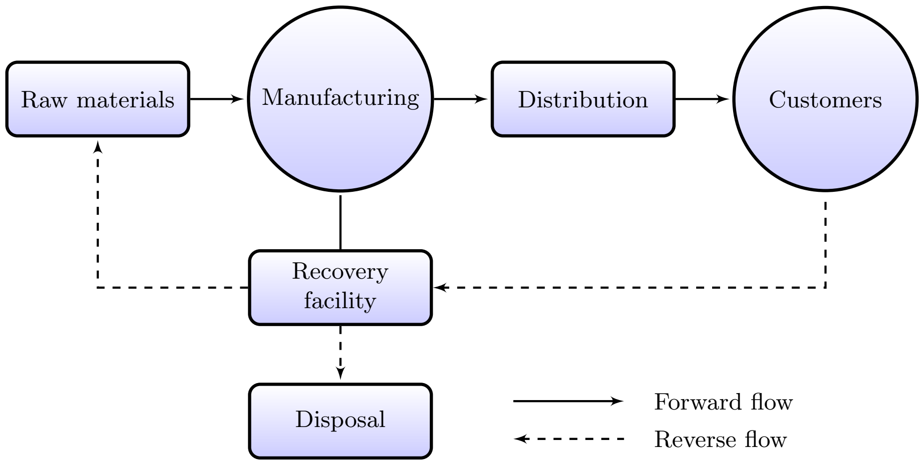


FIGURE 2.1: *A generic model of a closed-loop supply chain. Source: [38]*

The different phases in the life cycle of a product can be used to classify the type of closed-loop supply chain. There exists a manufacturing phase, a distribution phase, a phase of use and a end-of-life phase [26]. The supply chain closing at of the end-of-life phase will play the main part in this thesis. The actors involved in the recovery process, the type of units that are recovered and the form of re-use characterise a closed-loop supply chain. There is a difference whether the Original Equipment Manufacturer (OEM) carries out the re-manufacturing or a third-party becomes the re-manufacturer. Additionally, the collection process can be undertaken by various agents such as the manufacturer, a retailer or a third-party logistics service provider. This thesis will focus on the manufacturer carrying out the recovery process and the three actors as possible collecting agents. The type of product to be recovered can be either packaging, rotatable spare parts or consumer goods. The latter will be dealt with in this thesis [31]. Products that are returned within 30, 60 or 90 days after they had been purchased are classified as commercial returns. The replacement of a functional product by a technological upgrade, on the other hand, is an end-of-use return. A product that is technologically obsolete or of no utility to the customer any longer is referred to as an end-of-life return. The classification influences how time critical the collection process is, hence this thesis focuses on end-of-use or end-of-life returns [42]. Within the product recovery process there are five different options. Depending on the degree of disassembly that is required for the product, these options are repair, refurbishing, re-manufacturing, cannibalisation and recycling. In re-manufacturing used products are brought back to the quality standards of new products, in cannibalisation a small portion of the return is re-used and in recycling the material of the return is used again [86]. Therefore, re-manufacturing is a form of recovery that adds value whereas recycling focuses on the recovery of the material that had been part of the product. By rather re-using than discarding, re-manufacturing avoids waste [39]. This thesis deals with the options of re-manufacturing as the products brought back to the manufacturer will be turned into new products using the return's material.

According to Guide et al. [42] research on CLSC evolved from investigating on individual operations to take the entire closed-loop supply chain with a focus on reverse logistics into consideration. The evolution is described with the help of five phases of which each phase adds a new perspective to the topic.

In phase one, re-manufacturing and reverse logistics are treated as a technical problem. Re-manufacturing expensive assets was a critical concern to the US military that sponsored the early research in the 1990s. Increasing profitability of the re-manufacturing workshops by making operations work more efficiently was the main focus. Directives of the EU on end-of-life products as for example the directive on Waste of electrical and electronic equipment, the paper recycling directive and the end-of-life vehicle directive lead to companies searching for new ways to minimise the financial impact of compliance. Therefore, researchers investigated on the design for disassembly, design of networks for minimum-cost recycling and approaches to reduce the environmental impact. On one hand, the research on CLSCs focused on profit maximisation and thus was market-driven, on the other hand it centred around cost minimisation and hence resulted in a approach driven by the waste stream. Xerox pro-actively introduced a green-line of re-manufactured copiers leading to changes in the manufacturing-distribution network. This example pointed out the issue of value-creation within CLSCs. With this introduction Thierry et al. [86] identified the strategic importance of product recovery management. Future liabilities could be reduced by producing recyclable products. Moreover, discarded products now became the source of valuable components and material. Every company should analyse the opportunities and threats of the recovery of their product to uncover hidden potentials. Therefore, information needed to be gathered on the composition of the product, the return flow's magnitude and uncertainty, possible markets for re-used products and the current product recovery installations. Finally, eight managerial implications are highlighted. However, obtaining the necessary information stays difficult. Based on the technological feasibility, resources of returns that are sufficient, the existence of markets for recovered products and legislative prerequisites, the most profitable option is selected for the company. Setting re-use targets helps to measure performance of the actions taken. In most cases a redesign of the product is necessary. The cooperation between different actors of the supply chain is a requirement. In the development of reverse supply chains, opportunities to cooperate with competing actors exist. This conclusion should be emphasised as it already shows tendencies towards the application of a game theoretical approach in the context of reverse logistics. Additionally, manufacturing, operations and logistics management are influenced by product recovery management [2]. In conclusion, the developments of the first phase mainly centred around particular activities. However, prospects on future developments had already been present.

In phase two, the perspective started to shift towards process orientation. On the one hand, a classic operations research activity optimisation approach that is described in Dekker et al. [16] is applied. The topics of inventory control, reverse logistics networks, lot-sizing for re-manufacturing and the design of re-manufacturing processes are addressed, amongst others, in the new context of reverse logistics. Operations-research based tools for planning and control techniques within this field of research are introduced. On the other hand, the topic is explored through a business management view that connects the sub-processes to the topic. In 2000, Guide [39] emphasises the difference between re-manufacturing and traditional manufacturing operations. The characteristics are the uncertainty in quantity and timing, the uncertainty in quality, the necessary balance between returns and demand, the disassembly that is needed, the requirement for a network that can carry out the collection, the material matching restrictions and the variable processing time for re-manufacturing returns. Especially the challenge of greater variety demands different processes. With the help of a survey developed to assess manufacturing planning and control activities, the topic of defining an optimal channel choice between the re-manufacturer, a retailer and a third-party is identified as a future research issue. In 2001, Guide et al. [43] merges the general management perspective of Thierry et al. [86] and the aspects on operations research of Guide [39] into a business point of view on CLSCs that focuses on profitability [2]. Thereby, a framework that emphasises the economic potential of re-manufacturing activities is created. Bottlenecks in the process steps must be eliminated to access the profitability of CLSCs. The acquisition of a product determines whether re-manufacturing creates value or even increases profits. The Economic Value Added approach is applied to determine the potential profitability of re-manufacturing activities. Additionally, the facility design can be influenced by product acquisition and potential markets for re-manufactured products can be revealed. That is why, the perspective shifts from minimising cost to maximising profits in regards to product returns. The business perspective should explore the drivers for economic profitability and show managers how to influence these drivers. In 2003, Guide et al. [40] points out that a business approach needs to be adapted to the entire process. Practices that are suitable for forward supply chains can then be applied to reverse supply chains. A successful integration of the steps of the reverse supply chain forms a closed-loop supply chain creating opportunities for maximising profits. Hence, a life cycle approach for handling products is created. Thereby, the second phase introduced a duality to the operations research activity optimisation that is further investigated in Guide et al. [41] covering operations research based modelling approaches and the business economics perspective.

Phase three deals with the coordination of the reverse supply chain processes. The business economics approach connects to other aspects of operations research like game theoretical approaches. Typically, reverse supply chains are not controlled by a single actor. The different independent players involved in the reverse supply chain create an additional complexity in designing and coordinating the entire CLSC. Thus, game theory explores the strategic implications of the recovery processes. Contracting helps to coordinate the actions of the different players. With this new perspective, research started taking on a broader scope. Savaskan et al. [73] investigated in finding an optimal reverse channel structure. Game theory is applied to determine the best way to access the used products. That is why, the article of Savaskan et al. [73] as well as the articles following up on this approach create the foundation for the thesis. The analysis of Savaskan et al. [73], Savaskan et al. [74], Atasu et al. [3] and Chuang et al. [12] are described in detail in Section 2.3. Topics dealing with the improvement of component durability, the reduction of false failure returns, the effects of competition from third-party re-manufacturers and the reciprocity between new and re-manufactured products are addressed amongst others. Phase three moved CLSC from an extension of the supply chain knowledge to an acknowledged field of research.

In phase four the system is designed dynamically over the whole product life cycle. Aspects of volume, time sensitivity and quality of the returned products influence system design significantly. Depending on how time sensitive the product is, the CLSC has to be responsive to a high time sensitivity or cost-effective for a low time sensitivity. The interactions between the rate of collection, the durability and the life cycle of a returned product determine the design of the system by taking a look at the bigger picture of a CLSCs. The design requires an integrated perspective that also recognizes the different independent actors involved to make use of the full potential business value. The information on the various return types and the different time sensitivities need to be merged to create the maximum value over the entire life cycle of a product. The success of a business system is dependent on the right perspective at the time the system is designed.

Phase five additionally takes the valuation of the re-manufactured products and the behaviour of the customers in the market into consideration. In reality there are not only perfect product substitutes or secondary markets but a mixture of both extremes. Therefore, the fear of product cannibalisation might block recovery activities. Acknowledging this fact, research results show that re-manufactured products do not seem to cannibalise the sale of new products but can discourage low-cost product competitors. With the danger of cannibalisation the timing of introducing re-manufactured products into the market as well as the sales promotion become strategic decisions. Taking on an accounting point of view, re-manufactured products can either be regarded as a loss or as a potential source of profit. This view determines the willingness of a company to invest in recovery activities. In conclusion, other business disciplines like marketing and accounting are integrated in this fifth phase of the evolution of research on CLSCs [42].

Flapper et al. [26] describes different phases in the life cycle of a product. In each of these phases a closed-loop supply chain can be implemented. Aspects of business drivers, engineering, organisation, planning and control, information systems, environment and economics have to be taken into consideration to be able to choose in which phase the loop should be closed. Direct and indirect economic profits are identified as the major driver for future CLSC developments. Opportunities lie in enablers such as technical developments through increased design for re-usability as well as tracking and tracing of the location of products through RFID chips. However, these technological developments may also result in new challenges. An example is the use of lighter but less durable materials such as plastics to achieve a reduction in the fuel consumption of cars. CLSCs needs to focus on the business economics perspective to make further improvements. This will result in a greater recognition throughout the industry [42].

Souza [80] addresses strategical, tactical and operational issues in his review. The strategic decisions a manufacturer has to take is whether to re-manufacture or not. The manufacturer has to choose whether to use the strategic source of returns of trade-ins or leasing as well as how to respond to take-back directives imposed by legislation. Additionally, the manufacturer has to design the CLSC network and coordinate and incentive the members. A minor aspect deals with the impact on the design of new products. Tactical decisions are the strategy to collect products and the disposition of returns. These tactical issues are specified in the operational approach.

In a more recent review on the topic of closed-loop supply chains and reverse logistics provided by Govindan et al. [38] papers published from 2007 to 2013 are reviewed, categorised and analysed to create a foundation of past research and light on future directions. The article organises the diverse topics into designing and planning, price and coordination, the business perspective on CLSC and RL, manufacturing planning and inventory control, decision making and performance evaluation and the vehicle routing problem amongst others. Surveys try to find practical answers to scientific questions and different studies that analyses conceptually as well as qualitatively in subjects such as product life cycle management. Especially the consideration of uncertainties is pointed out. Different parameters chosen as non-deterministic are analysed in non-deterministic approaches. Thereby, the importance of analysing different data sets of CLSC and RL networks is emphasised.

Looking at different publications the connection can be drawn that game theory approaches are usually applied to pricing and coordination problems. The methods used in game theory approaches are mainly analytical. Additionally, simulation techniques seem to be applied in many different cases. Linear modelling approaches dominantly solve design and planning problems. Future opportunities are highlighted in particular. Studies should integrate green aspects and sustainability into CLSC and RL research and not solely try to prove which aspect covers the topic. A comprehensive view on the topic combining all the special subjects previous studies have concentrated on should be worked out. Regarding the topic of considering uncertainties, non-deterministic approaches should be modified by including approaches such as fuzzy logic, interval approaches and chaos theory. Staying with stochastic approaches to face uncertainties, these should be extended to two-stage approaches with robust optimisation techniques. Additionally, the technique of forecasting should be included to address uncertainty issues. In general, it is important to investigate on all parameters that may lead to uncertainty. Furthermore, there are opportunities to improve modelling approaches. Non-linear programming approaches as well as convex optimisations are changes that can to lead towards realistic models in the future, since linear programming models do not reflect the complexity of real-world problems. Not only the modelling approaches but also the methodology behind them can be extended. Especially as the problem becomes more complex, heuristics or meta-heuristics become an effective way to find good feasible solutions. A combination between exact and heuristic could show a future direction for research in the CLSC and RL environment, since analytical solution approaches are closer to theoretical solution methodologies. The incorporation of simulation into analytical game theory approaches can be a step towards this direction to create a realistic but still solvable situation. Operational decision variables could be integrated with tactical and strategic variables. The decision variables should be constantly updated to reflect the current development. Finally, single-objective approaches should be turned into multi-objectives to resemble the real-world. Therefore, green, sustainable, environmental and resilience objectives need to be included. These outlooks given by Govindan et al. [38] on the future will provide ideas for the research of this thesis.

2.1.2 Characteristics of reverse logistics network

Comparing forward to reverse logistics, the general assumption that moving parts from A to B is similar to moving them from B to A is more true if the collection is outsourced to a third-party logistics service provider. Small differences between distribution and collection as well as inbound and outbound transport may be observed. Taking back end-of-life returns is less time-critical than new product deliveries and thus leaves more time for optimisation of vehicle routes and full truck shipments. Depending on the collection network, a great number of stops per tour may occur in reverse channels as the number of simultaneous sources is much smaller than the number of demand locations in forward channels. As the differences seem rather limited, the challenge of a combination of both channels arises [27].

Various approaches lead from complete integration to total separation. Even in a closed-loop environment different volumes, different timing and different requirements of handling are arguments towards a separation of forward and reverse transport. Additionally, vehicle loading restrictions of rear-loaded trucks that require first-in-first-out load access may lead to delivery and collection stops that cannot be mixed. Therefore, expected benefits of combination may be traded off against investment in specialised vehicles. Due to these reasons, this thesis focuses on the transport of returns. Nevertheless, the question whether to integrate reverse logistics into the design phase of a closed-loop supply chain or extend the existing forward supply chain after the implementation will be addressed in the following paragraphs [27].

Over time, the definition of reverse logistics has been changing. In the beginning, the meaning simply expressed going the other way. An emphasis of the environmental aspect followed with the definition of the Council of Logistics Management [15, p. 4] in 1992 as "the term often used to refer to the role of logistics in recycling, waste disposal, and management of hazardous materials; a broader perspective includes all relating to logistics activities carried out in source reduction, recycling, substitution, re-use of materials and disposal." Afterwards the focus on direction was introduced again. With the overall goal and the processes of RL pointed out, RL was defined according to Rogers et al. [70, p. 2] as "the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal". Finally, leading to today's definition that keeps the essence of the definition of Rogers et al. [70] without specifying on a point of consumption or origin to give it a wider scope [15].

In 1997, Fleischmann et al. [31] reviewed the first developments in the topic of reverse logistics from a quantitative point of view. The motivation for recovery can be ecologically as mainly found in European countries but also economically as in the US' focus. The concept of sustainability tries to combine these motivations. The first category deals with distribution planning aspects that describe the collection and transport from customers back to the manufacturer. Therefore, this category is the most applicable to give a background to this research. Inventory management and manufacturing management create the second and the third category. For the distribution of returns in the forward channel, a separate reverse channel or a combination of both channels can be used. The actors of the reverse channel have to be identified, the functions have to be described and the relation between the forward and the reverse channel has to be taken into consideration to determine the best option. An integration of forward and reverse channels was still a direction for future research at that point in time. Although some approaches towards this direction had been made already in the network design stage.

The paper by Kroon et al. [51] promotes a separate modelling of reverse flows. The practical application considered in this paper is the re-use of secondary packaging material. Methods to create a return logistics network for returnable containers is created and applied to a case study. Therefore, a first quantitative model to plan a reverse logistics network is presented. The system is simulated at first, followed by an optimisation approach. The dependence of the system's success on economic implications is emphasised especially as some investments have to be made.

The Reverse Logistics Executive Council [70] examined practices and determined trends of reverse logistics by companies about their RL. Not only the return of products but also the return of packaging is analysed. The activities examined are the process of collecting end-of-use products from the customer and choose from a variety of options such as re-sell, re-manufacture or recycle according to the condition of the product. For packaging, there are the options of re-use and salvage amongst others. The most important question is how to get the returns from the customer to the facility. Recognizing the strategic potential of reverse logistics is a future trend. However, there are many problems specific to reverse logistics like the lengthy cycle times for processing. Approaches to overcome these difficulties are standardising processes, outsourcing to a third-party with expertise as well as a combination of different strategies to handle returns.

Dowlatshahi [17] identifies five streams of research in a review on the topic. The first group of papers focuses on global concepts of reverse logistics. Important factors that are identified are the compatibility of current manufacturing and re-manufacturing processes, an analysis of the cost and benefits of recovery, a bill-of-material that is restructured according to the new requirements, the effective management of organisational procedures of RL, an integration of transportation modes and a thought-through packaging for RL. Quantitative models are addressed in the second group of articles. Papers dealing with transport, warehousing and distribution are grouped together to describe a third stream. Company profiles form the forth group. The fifth group concentrates on applications for RL. In conclusion, most of the articles are practitioner-orientated and lack in describing the basic structure of a reverse logistics system.

Reverse logistics creates a link between the supply of returns and the demand for re-usable products. This link includes the collection, the testing and sorting, the re-manufacturing and the redistribution. A closed-loop network may evolve from the coincidence of two market interfaces for used and re-usable products [30]. But how does the reverse channel differ in detail from the traditional forward channel?

According to Tibben-Lembke et al. [87] reverse logistic flows are more difficult to forecast than forward flows. However, trends that are applicable to forward flows can also be used for reverse flows to some extend. Therefore, information based on forward supply chains benefits reverse supply chains as well. Instead of a one-to-many distribution, RL has to deal with a many-to-one distribution for the various actors involved. A combination of forward and reverse flows should be taken into consideration to make use of the full capacity of a truck. While products and packaging are of uniform quality in forward supply chains, they may vary significantly in the reverse channel. Additionally, the importance of time is another difference. However, whether a return is time-critical or not is determined by the product itself. These factors amongst others highlight the differences between forward and reverse logistics. Especially the enormous cost of collection and transport is a major obstacle in RL. A group of businesses take ownership of the returns. After the collection from the customer different actors sell the products back to the manufacturer. Depending on the channel members' abilities to recover the returns, the reverse logistics network can take on different structures [23].

Strategic decisions on the choice of collection, the operating facilities and the transport links need to be taken. In contrast to traditional forward logistics, the factors of supply uncertainty and interdependence between forward and reverse flows are fundamental to RL. Not the demand of new products but the supply of used products is the main unknown in RL. The returns form a less standardised input than raw materials or component supplies. The major challenge lies in effectively matching this supply to the demand for used products that directly influences manufacturing cost. Incorporating the synergy effects of forward and reverse flows is a great potential in the creation of closed-loop supply chains. An opportunity to attain economies of scale is the transport as distribution of products can be combined with collection of returns to reduce empty truck rides. However, closed-loop supply chains are generally realised by sequentially adding reverse channels to the existing distribution structure. This approach might raise compatibility issues. Therefore, it should be evaluated whether to redesign the entire closed-loop supply chain network or not to achieve optimal performance.

The comparison realised by Fleischmann et al. [30] between the two-stage approach of designing the forward and the reverse network sequentially and the integral design of optimising both parts of the network simultaneously reveals that the integral approach leads to a completely different network structure. Nevertheless, the cost of both approaches are almost similar in the numerical example that was carried out to compare the two cases. Even changes in the set of parameters lead to similar results. The fact that in terms of volumes and cost the forward supply chain outweighs the reverse channel explains this outcome. That is why, the overall optimal solution is close to an optimal solution of the forward channel. From a business perspective this means that including a reverse channel does not necessarily mean redesigning the existing logistics network fundamentally. The integration of a reverse channel becomes less complex, hence easier to implement into the organisational structures and less costly. Only if the reverse supply chain has a major impact on the supply of material, the existing supply chain might move towards the reverse flows. Additionally, differences in labour cost might change the overall logistics network as new products are substituted by re-manufactured ones with different cost drivers.

At the moment, the choice of the strategy of collection is further examined. There are two collection strategies as pointed out by Aras et al. [1]. On the one hand, in the pick-up strategy products are directly collected from the customers. On the other hand, the customers undertake the travel effort to bring their returns to a collection point in the drop-off strategy. The fixed cost associated with operating a facility and the variable cost for collection and transport must be carefully assessed to choose the strategy that is most suitable. Additionally, the metrics of the goods as well as their nature has to be taken into consideration. In the drop-off system, the incentive for the customer needs to be high enough to enhance the customer to travel to the next collection point. Thereby, the customer creates a reverse logistics network. Nevertheless, both strategies of collection have to answer the question of the cost of collection that originates from shipping the used product from the customers to the recovery facility. With direct collection, the costs and influencing parameters can be observed. In indirect collection the cost has to be incorporated into the customers incentive, thus making an investigation on the logistics network difficult. That is why, this thesis will focus on the strategy of directly collecting used products from customers through different reverse logistics channels.

2.2 Theory of games

"Game Theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" according to Myerson [56, p. 1]. Choices made by individuals within a group can affect the entire group of people. Game theory thus focuses on the interdependence between the individuals. It provides a tool to analyse the interactions between strategically acting agents [18]. The study of conflict uses abstract common strategic features to analyse theoretical models. The field of research is given the name "Game Theory", since the patterns that are applied originate from actual games such as poker [49].

A pioneering analysis of imperfectly competitive markets is the publication of Cournot in 1838. Cournot studied the problem of companies in a market competing simultaneously over the amount of output to be produced. The model is defined by the number of decision-makers involved in the competition. The generalisation of this model deals with a small number of sellers dominating a market called oligopoly for which Cournot developed a method of analysis. In 1913, the development was followed by Zermelo who looked into the game of chess. Zermelo stated that one of the two players playing chess has a winning strategy from any position on the board. That is why, chess has always a solution. In addition, the procedure of backward induction was introduced by Zermelo. Borel followed by actually defining the games of strategy for the first time in 1921. However, the publication "Zur Theorie der Gesellschaftsspiele" of the mathematician Von Neumann [63] that investigates on an approach to achieve optimal results playing parlour games in 1928 and the subsequent book "Theory of Games and Economic Behaviour" [91], released in collaboration with economist Morgenstern in 1944, mark the beginning of the interdisciplinary field of game theory. For the first time, the mathematical theory of games of strategy was applied in economics. The formalisation of the concept of a game as well as three other important contributions were made. The first one is the expected-utility maximisation theorem based on axioms. This was followed by an optimal solution of two-player zero-sum games. In a zero-sum game one player's gain is the other players loss. Therefore, the zero-sum game is used as a foundation of Von Neumann and Morgenstern's mathematical analyses. Extending this analysis further leads to the introduction of cooperative games. This extension is the final contribution, yet only acts as a starting point in the analysis of cooperative games in general [18].

In 1950, Nash [58] introduced his concept of equilibrium in non-cooperative games. The idea of a Nash equilibrium results from every player acting on the correct assumptions of the opponent's action. Therefore, strategy choice depends on the behaviour of the opponent. If no player has an incentive to change the choice of action, the game is in equilibrium. Given this strategy choice background, the Nash equilibrium approach advanced the analysis of games from a focus on zero-sum games to non-zero-sum games. Additionally, Selten [75, 76, 77] generalised the Nash equilibrium approach to dynamic games in 1965 and 1975. In Selten's paper from 1967, he deals with the problem that changes in pricing do not necessarily result in time synchronised reactions in demand. Reactions might be delayed, since the market position of a company does influence the demand for a product in the same way. Therefore, the present actions of a player become increasingly important for future consequences. Taking these conclusions into consideration, the paper from 1975 re-exams the definition of a perfect equilibrium point from a general point of view to a subgame perfect equilibrium perspective. Furthermore, from 1967 to 1968, Harsanyi [44, 45, 46] applied the Nash equilibrium approach to settings in which players have incomplete information about the opponents' preferences or choices. The first part of Harsanyi's deliberations describes a basic model in which players are uncertain about the payoff functions of the opponents. The idea of a Nash equilibrium is then transferred to the newly described situation. Finally, the third part of the article investigates on the probability distributions over the alternative possibilities of the uncertainties. Game theory moved another step further towards formulation of real-world problems with this more realistic underlying assumption of incomplete information [18].

Generally, there are two basic assumptions about the players of a game. The first one is that players are rational and the second one is that players are intelligent. If a player constantly makes decisions in pursuit of the own objectives, the player acts rational. The assumed objective of each player is the maximisation of the expected value of profit measured in some utility scale. Building on fundamental results of decision theory with an extension to several assumptions, Von Neumann and Morgenstern developed the Neumann-Morgenstern utility theorem. According to Myerson [56, p. 3] the key assumption is that "if a decision-maker would prefer option one over option two when event A occurs, and he would prefer option one over option two when event A does not occur, then he should prefer option one over option two even before he learns whether event A will occur or not". The theorem states that under certain weak consistency axioms of rational behaviour there exists a way to assign utility numbers to various possible outcomes in to choose the option that maximises a player's expected utility. As utility values do not need to be measured in a currency, maximising the expected utility payoff has not necessarily monetary value. Problems of the interaction between rational decision-makers must be analysed simultaneously, since the rational solution of an individual's problem can only be solved by taking the solution of the other individual's problem into consideration. If a player knows everything about a game and can make inferences about the situation of the opponents, the player is considered to be intelligent. Therefore, each player in the game needs to understand the theory that describes the behaviour of intelligent players in a certain game and must make predictions about it [56].

The analysis of games has two goals. The first one is the description of the competitive situation. The second goal lies in advising the players on the best way of playing a game [55]. Thereby, different solution concepts are presented to the arising conflict. Both goals will be described next to be addressed in the analysis of the situation of conflict modelled in this thesis.

The analysis of a game begins with the specification of a model that describes the situation of conflict. With increasing levels of abstraction the extensive and the strategic (or normal) form are the most important representations of a game.

The extensive form of a game is a literal translation of the rules, thus forming the first level of abstraction [49]. The rules of a game specifies the group of players, the actions and alternatives available to each player, the order in which each actor gets to play and the gain or loss derived by the player's choice of action [18].

The definition of the extensive form introduced by Kuhn [52] is now a standard in most of the game theory literature and defines a general n-person game Γ_e in extensive form as a game tree K with the subsequent specifications.

1. The player partition P refers to dividing the moves up into $n+1$ indexed sets $P_0, P_1, \ldots, P_n$. The moves in P_0 are called chance moves, whereas the moves in P_i are called personal moves of player i for $i = 1, \ldots, n$.
2. The information partition is defined by a separation of the moves into sets U which is a refinement of the player and alternative partitions (each U is part of $P_i \cap A_j$ for some i and j) such that no U contains two moves in the same play. The sets of the information partition are called information sets.
3. For each $U \subset P_0 \cap A_j$ a probability distribution over $1, \ldots, j$ is introduced, assigning a positive probability to p. These information sets are assumed to be of one element.
4. The payoff function h assigns a n-tuple of real numbers $h(W) = [h_1(W), \ldots, h_n(W)]$ to each play W.

It can be effectively used to translate a verbal description into a game, since the extensive form is a pictorial representation of the rules. It has the form of a game tree that is made up of a root and branches resembling the players, the choice of actions and the payoffs. The combination of one pure strategy from each player's list leads to the strategic or normal form of a game. Especially with two-player games, a game can be represented as a matrix. This representation is conceptually simpler and thus more convenient for the purpose of mathematical analysis. However, most real-world problems deal with more than two players and can involve mixed instead of pure strategies assigning probabilities to certain outcomes. The strategic form provides a general description to enable the analysis of these games. The specification of the set of players, the set of options per player and the link between a player's payoff and the options the players may chose from is needed to define a game in strategic form [18, 49].

According to Myerson [56] a game in strategic form is a general n-person game Γ_s of the form

$$\Gamma_s = (N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}), \tag{2.1}$$

where N is the set of players in the game Γ_s. For each player $i \in N$, A_i is the set of all available strategies to the player. That means each player i must choose one of the strategies in the set A_i when the strategic form game Γ_s is played. A combination of strategies that the players in N might choose is a strategy profile. The number U_i represents the expected utility payoff for any strategy profile $a = \{a_j\}_{j \in N}$ in A that player i would get if a was the combination of strategies that had been played.

In contrast to the extensive form, the strategic form is a static approach to model a game. It is assumed that all players choose their strategies simultaneously and that the question of timing is being ignored. However, with repeated games for example, each repetition can be presented as a play of the game in strategic form. Additionally, more alternative actions can be added to the strategic form of a game, if for example pre-game communication between players becomes available. Therefore, the strategic from will be used to analyse conflict situation between the players in the course of the thesis [56].

So far the players with their payoffs as the gains and losses that can be attained by choosing an action have been explained, now the focus falls on the strategies. At each decision point, a player must make a choice. Thus, the strategy of a player determines what to do. A decision based exactly on one action is called pure strategy. Mixed strategies, on the other hand, are a probability distribution over pure strategies. Regulated by chance, a player might decide between pure strategies to make a game solvable. Therefore, the mixed strategy for a player is a probability vector [18, 49].

All games are classified by the sum payments the players receive at the end of a game. No manufacturing or destruction of goods is involved. If the sum of all payments is zero, the type of game is called zero-sum game providing a general understanding of a game. However, the sum of all payments is generally not zero or sometimes not even constant in economically significant schemes. Therefore, this type of game, that will also be described in this thesis, is referred to as a non-zero-sum game [91].

The thesis deals with different actors that compete over carrying out the collection of used products from the customer to the recovery facility. Therefore, the game will be described in strategic form. The rate of collection can take on many different levels, hence the players do not only decide between "collection" or "no collection", but can set the rate of collection according to each player's needs. Finally, as in most economic situations, the sum of payoffs will not equal zero. The profit made by collecting returns will determine the outcome of the n-player non-zero-sum game. The following subsections are dedicated to solution concepts for these types of games, since the game can exclude or include cooperation amongst players.

2.2.1 Non-cooperative games

Zero-sum games are always non-cooperative, since cooperation does not benefit any of the players. Therefore, the element of cooperation is not present in zero-sum games. In contrast, the possibility to cooperate with any or all of the opponents is present in non-zero-sum games. Nevertheless, if both players make their decisions independently and simultaneously before each play, a non-zero-sum game can also be non-cooperative. Without cooperation, pre-play communication is forbidden and payoffs are distributed amongst all players according to the predefined rules of the game. Examples for non-cooperative games are bitter rivals in a business competition or large companies in a market competition with antitrust legislation restricting any cooperation [49, 55, 85].

The solution concepts for non-cooperative non-zero-sum games differ slightly. Zero-sum games and non-cooperative non-zero-sum games are similar as both incorporate the aims to maximise the security level and tend towards strategies that result in equilibrium. However, especially the fact that non-zero-sum games can have more than one equilibrium point, requires an extension of the concepts solving non-cooperative non-zero-sum games. Therefore, these solution concepts will be analysed carefully next [85].

The well-known prisoner's dilemma introduced by Tucker [88] is used as an example to explain the characteristics of non-cooperative non-zero-sum games. Two men held separately by the police are suspected of a joint violation of law. The district attorney believes that these two men committed an armed robbery that he has been unable to clear up so far. However, his suspicions are not proven adequately by the evidence. Therefore, the district attorney approaches each suspect to confess and give evidence against each other. He states that if one confesses and the other one rejects, the former will be rewarded with zero years in prison and the latter will be charged with six years. The two prisoners know that if both of them confess, they will serve five years in prison and if both of them deny, the robbery charge will be dropped but each of them will get one year in prison for carrying a concealed weapon.

Generally, the normal form of a two-person game can be represented by a matrix of pairs.

Each entry of the bi-matrix is an ordered pair of numbers. The payoffs of Player 1 (P_1) and Player 2 (P_2) are the numbers of the ordered pairs. P_1 is referred to as the row player, while P_2 plays the columns. The situation of the prisoner's dilemma translates to the following bimatrix of a two-person game.

		Player 1	
		Confess	Defect
Player 2	Confess	$(5,5)$	$(0,6)$
	Defect	$(6,0)$	$(1,1)$

For each prisoner the pure strategy "confess" clearly dominates the pure strategy "defect". A strategy pair is in equilibrium if none of the players can gain by deviating from this particular strategy as long as the strategy of the opponent remains fixed. Therefore, a unique equilibrium point is given by the pure strategy "confess". Individual rationality implies that both players should confess the crime. Any deviation from this strategy might result in a six year sentence, while the partner gets released immediately. However, both prisoners would be better off, if they would deny simultaneously. The payoff pair $(1,1)$ dominates the payoff pair $(5,5)$ from the viewpoint of collective rationality. Whether to trust the other player or not is the dilemma of this game. That is why, the way the game is played depends on whether a player operates on individual or collective rationality [18, 49].

Games with pure strategies are solvable through determining the dominant strategies. A pessimistic estimate of how much a player can attain can be computed with the help of the "maximin value" in games with pure and mixed strategies. In a two-person game, the maximin value v_1 for player P_1 is calculated under the assumption that player P_2 will make a choice trying to minimise P_1's payoff π_1. With the probability vector $\boldsymbol{p}$ as P_1's mixed strategy and $\boldsymbol{q}$ as the mixed strategy of P_2, the maximin value v_1 of P_1 is computed by

$$v_1 = \max \min \pi_1(\boldsymbol{p}, \boldsymbol{q}). \tag{2.2}$$

In the case of a non-zero sum game, player P_2 rather tries to maximise the own payoff. Hence the action chosen might not be the one that minimises P_1's payoff. However, the procedure puts a lower bound or security level on P_1's payoff and is easy to calculate as P_1's payoff matrix is regarded as a zero-sum game. If P_2 minimises P_1's payoff, P_2 would try to maximise the negatives of those payoffs and thus acts as the column player of the game. After eliminating dominated rows and columns, this procedure easily reveals an optimal strategy for P_1. Besides, P_2's maximin value v_2 can be computed by solving the game of the transposed matrix that carries the negative entries of the original matrix game [55].

In general, computing equilibrium pairs is rather difficult. A payoff pair is in equilibrium, if no player has an incentive to change the action based on the correct assumption about the opponent's behaviour. In non-cooperative two-person non-zero-sum games, the pair of mixed strategies $\boldsymbol{p_1}$ and $\boldsymbol{q_1}$ is in equilibrium given that

$$\begin{aligned} \pi_1(\boldsymbol{p}, \boldsymbol{q_1}) &\leq \pi_1(\boldsymbol{p_1}, \boldsymbol{q_1}) \quad \forall\ \boldsymbol{p} \text{ for player } P_1 \\ \pi_2(\boldsymbol{p_1}, \boldsymbol{q}) &\leq \pi_2(\boldsymbol{p_1}, \boldsymbol{q_1}) \quad \forall\ \boldsymbol{q} \text{ for player } P_2\ . \end{aligned} \tag{2.3}$$

In addition, there exists a graphical method that is yet only applicable for two-person games in which the players only have two pure strategies [55].

A non-cooperative payoff region takes all possible pairs of mixed strategies of a two-person game into consideration. The responding payoffs pairs can be plotted in a Cartesian coordinate system with π_1 as the horizontal and π_2 as the vertical axis. Thereby, the dominance between payoff pairs can be read from such a plot as the dominating payoff pair is north-east of the dominated pair. A payoff pair (u, v) dominates another payoff pair (u', v') given that $u \geq u'$ and $v \geq v'$. A payoff pair that is not dominated by any other pair is Pareto optimal.

In the prisoner's dilemma, the payoff pair $(5, 5)$ is not Pareto optimal since it is dominated by $(1, 1)$. However, the equilibrium pair is where both players confess the crime. If equilibrium pairs are interchangeable and equivalent a game is solvable in the Nash sense. As the prisoner's dilemma only has one equilibrium pair it is solvable in the Nash sense. The Nash equilibrium is one of the widely used solution concepts in non-cooperative game theory [55].

There is a great range of situations of conflict that are similar to the prisoner's dilemma. The situation of conflict in this thesis will also be modelled as a non-cooperative game. The common features of these games are that players would do well if they would cooperate, but neither player trusts each other. If only one player cooperates and the opponent does not, the cooperator does badly and the defector benefits from choosing the non-cooperative strategy. So far the prisoner's dilemma was only played once. What would happen if the game was played repeatedly? Is it more likely for cooperation to arise in repeated games?

2.2.2 Games with cooperation

In non-cooperative non-zero-sum games, the value of a game may differ for a player, since the player might obtain different payoffs for different equilibria. Therefore, each player can have a preferred equilibrium. However, the choice of equilibrium of one player might have consequences for all other players. Therefore, the possibility to harmonise the decisions of the players arises, especially if a game is played repeatedly. This other type of non-zero-sum game is a game with cooperation amongst the players. Before the game starts, the players are allowed to make binding agreements on how to play during the game. The strategies can be coordinated to increase payoffs, hence players decide whether to take part in a coalition or not. Another agreement specifies how players divide the jointly acquired payoffs among members of a coalition. These agreements should induce other players to join a coalition. If the payoffs are non-transferable, side-payments in form of monetary units are a feasible alternative. Especially pre-play communication provides agreements on the coordination of strategies and the sharing of payoffs [49, 55].

If games are played repeatedly, players might behave differently towards their opponents then when a game is only played once. Collective rationality becomes more likely to operate. In a repeated game, there is a finite or infinite sequence of rounds in which players get information and make their decisions. A player needs to take the effects of the current move in the future and the information about the other players into consideration, since no move is necessarily the last one. Therefore, the previous stage of a game can be used to determine the move of the player in the following stage, since the repeated encounters give players the opportunity to cooperate. Reputations can be developed by players in repeated games. Additionally, threats can be used to influence the action of other players in case of dependency [54, 56].

Every repeated game consists of a component game, the stage game G, that is repeated a number of times T. A finitely repeated game has a fixed end T, whereas an infinitely repeated game does not. In strategic form, the stage game $G = (N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N})$ includes N as the set of players, the set of strategies A_i available to player i and the payoff functions u_i [18, 50].

The prisoner's dilemma is analysed exemplary to explain the dynamics of repeated games. The choice to either "confess" (C) or "defect" (D) is taken by the prisoners simultaneously. If the game is repeated once, the choices are revealed and the prisoners have to make another decision. The payoffs are computed by the sum of the payoffs in both stages of the game.

The finitely repeated game used in the first example has five subgames described by four round-two interactions. Within each subgame of the original game, the subgame perfect equilibrium strategies must locate a pair of actions that form a Nash equilibrium. That is why, each player's strategy must specify the action (C) in round two, if we look at the second game through backward induction. In round one there are only two strategies to play, either choose (C) or (D) respectively, followed by no matter what but (C) in the second stage. Choosing between these two actions is as if the players would only play the game once. Therefore, the Nash equilibrium of the first example is (C,C). Due to the subgame perfect equilibrium strategies, the players will always choose (C,C) if the prisoner's dilemma is only repeated once.

The second example works with a slightly modified version of the prisoner's dilemma. Assuming the prisoners had the opportunity to communicate. Therefore, a third strategy, the strategy to partly confess denoted trough (P), arises. The stage game G is played T times allowing every player to take one of the three actions (C), (D) and (P) in every round. After each stage game the choices are revealed to the prisoners and they get to make a new decision for the next round. The sum of the payoffs gained in every stage game G forms the payoff per player.

The players continue making their decisions according to the rules of prisoner's dilemma, but the game is repeated an unlimited number of times in the third example. The game is referred to as the infinitely repeated prisoner's dilemma, since it has no fixed end. The probability that the same players play the stage game again is expressed through δ and the probability that the current game is the last one is denoted by the normalisation factor $(1 - \delta)$. At the t-th stage every player i gets a payoff π_{it}. The chance that the t-th stage of the game ever gets played is δ^t. Therefore, the expected payoff at the t-th stage is $\delta^t \pi_{it}$. The total expected payoff is the sum of the expected payoffs per stage game.

In conclusion, the number of repetitions of a stage game G can either be finite or infinite. The total payoff of a finitely repeated game is computed as the sum of payoffs at every stage, whereas the total payoff of a infinitely repeated game is worked out by summing up the discounted expected payoffs. Players start to take long-term gains in addition to short-term payoffs into consideration. Players might even be willing to sacrifice short-term gains, if a player is convinced that the action will result in a reciprocal reward in the future. Hence, these developments induce the communication and maybe even cooperation between different decision-makers [18].

If expected payoffs can be increased by a correlated or jointly randomized strategy the players might want to communicate with each other and coordinate their moves. The players might even introduce contractual agreements to transform a game. A contract of the undersigned promising that player P_1 chooses action one and player P_2 chooses action two, if the contract is signed by both players or that player P_1 chooses action three or four if the contract is only signed by player P_1. This is an example of contractual agreement. Another example is the specification of a contract that, if signed by both players, a coin will be tossed to determine the implementation of the player's actions. The great variety of bargaining and signing contracts as explicit options in a strategic-form game would make the list of actions unmanageably complicated. Solution concepts that express the impact of implicit communication opportunities can be added to the strategic options to facilitate the illustration of possible agreements between the players [56].

In two-person cooperative non-zero sum games, binding agreements are made before the game begins. The players choose a joint strategy to increase the individual payoff, but do not share payoffs or side-payments amongst each other. The nature of the games restricts the sharing of payoffs. In the prisoner's dilemma for example, each suspect has to serve his own sentence.

The joint strategy of the prisoner's dilemma would be to make a binding agreement on denying together. The joint strategy assigns a probability to each pair of pure strategies. In comparing the cooperative payoff region to the non-cooperative, a larger set becomes available to the players. Hence, more payoff pairs become obtainable to players that cooperate. A cooperative payoff region is always a convex set that is closed when plotted in a Cartesian coordinate system. Each player considers the payoff from the joint strategy to be at least as high as the payoff from the maximin value to make an agreement on which joint strategy to apply. In addition, the joint strategy should be Pareto optimal. Therefore, the bargaining set of a cooperative game contains the Pareto optimal pairs (u, v) such that

$$u \leq v_1 \, \text{and} \, v \leq v_2 \ , \tag{2.4}$$

where the maximin values of P_1 is v_1 and v_2 denotes the maximin value of P_2 [55].

A bargaining situation between two actors evolves if both individuals have the opportunity to cooperate, although only one player might actually gain from the cooperation. Examples for such situations are state trading between two nations or negotiations between an employer and a labour union. The bargaining model introduced by Nash [57] attempts to establish a fair method of choosing a payoff pair in the bargaining set. Presented with a payoff region R and a status quo point $(u_0, v_0) \in R$ the arbitration procedure Ψ produces a payoff pair, the so-called arbitration pair, that is fair to both players. The six axioms of individual rationality, Pareto optimality, feasibility, independence of irrelevant alternatives, invariance with respect to utility transformations and symmetry need to be satisfied. The arbitration procedure Ψ can be divided into three sub-processes. Firstly, for a given pair of threats the players agree to an outcome that satisfies the six axioms. The payoff region is described by the player's payoffs. Each player computes the maximin values resulting in the status quo point. The threat strategy leading to the status quo point is chosen next. Therefore, the payoffs that satisfy the six axioms are determined in the third and last step. The arbitration pair is among the payoff regions' points that dominate the status quo point. Finally, the maximum of the function on this line segment determines the arbitration point [49, 55].

What if players are able to fully cooperate amongst each other resulting in a share of payoffs? Then coalitions of players cannot only make binding agreements on the coordination of strategies but also on pooling the individual payoffs and redistributing the total of payoffs in a specific way. The set of players do not necessarily have to be greater than two. However, in the case of a two-person game this analysis may be trivial. The theory on cooperation in games can be applied to non-zero-sum as well as zero-sum games.

A coalition is a subset of the set of players that forms to agree on coordination and division of payoffs. The set consisting of all players N is denoted by $\mathcal{P}$ and called the grand coalition. The counter-coalition of a given coalition $\mathcal{S} \subseteq \mathcal{P}$ is $\mathcal{S}^{\mathrm{c}} = \mathcal{P} - \mathcal{S}$. Hence, the counter-coalition to $\mathcal{P}$ is the empty coalition $\emptyset$. In a game with N players, generally 2^N coalitions can form. The questions that arise from the possibility of cooperation deals with which coalitions are likely to form and on how to divide the payoff amongst the members of the coalition. The game can be analysed as a two-player non-cooperative game, since it is a competition between the coalition $\mathcal{S}$ and the counter-coalition $\mathcal{S}^c$. Even with the formation of the grand coalition and the empty coalition, this is applicable. This abstraction does not only simplify the description of the situation of conflict, but also facilitates the analysis. The members of each coalition would choose a joint strategy that allows them to gain at least the maximin value. Therefore, the characteristic function form denoted by $v(\mathcal{S})$ consist of the maximin value of the coalition $\mathcal{S}$. The largest payoff that all players can achieve is the characteristic function form of the grand coalition with the value of $v(\mathcal{P}) = 1$. By definition, the value of the characteristic function form of the empty coalition is $v(\emptyset) = 0$. In general, a game in characteristic function form must consist of all players $\mathcal{P}$, defined for all subsets so that $v(\emptyset)$ is empty and superadditivity holds that describes the gain from cooperation. With the help of these definitions, the relative strength of a coalition can be estimated [55].

How should the payoffs be divided amongst all members of a coalition? An imputation is a possible distribution of the payoffs available. Two solution concepts are widely discussed. The first one is a stable method named the Core and the second one is the Shapley value that is a fair method of payoff division.

The Core holds all imputations that are not dominated by any other imputation through any other coalition. Problems with the Core are that most of the time it is either empty or has so many imputation that it becomes difficult to evaluate which ones are likely to occur. Therefore, the solution concept of the Shapley value is described in detail.

The concept introduced by Shapley [78] takes the player's contribution to the success of the coalition into account. If $\mathcal{S}$ is the coalition a player belongs to and the characteristic function of the game is denoted by v, then the measure of the amount that P_i has contributed by joining $\mathcal{S}$ is expressed by

$$\delta(P_i, \mathcal{S}) = v(\mathcal{S}) - v(\mathcal{S} - \{\mathcal{P}_i\}). \tag{2.5}$$

It is assumed that the grand coalition forms, since the players collectively agree on an imputation. The process of formation starts with one player being joined by a second player, they are later joined by a third player and so on. An ordered list of players with the k-th player in the list being the k-th player to join characterises this process. Various orders are possible. In general, there are $N!$ possibilities, since the order is assumed to happen at random. Therefore, a probability of $\frac{1}{N!}$ is assigned to each of the possibilities. A general definition of the Shapley value ϕ_i for P_i is to make the same sort of calculations for every possible order of players. Thereafter, each player is weighted by the probability $\frac{1}{N!}$ of the order that occurs. Finally, the results are summed up and each player gets a payoff reflecting the amount of contribution. There are $(N-k)!$ permutations of players coming after P_i and $(k-1)!$ permutations coming before. The number of players in coalition $\mathcal{S}$ is denoted by $|S|$ together with the ordering and the contribution per player resulting in the Shapley value ϕ_i calculated by

$$\phi_i = \sum_{P_i \in \mathcal{S}} \frac{(N - |S|)!(|S| - 1)!}{N!} \delta(P_i, \mathcal{S}). \tag{2.6}$$

The Shapley vector for v is an imputation and thus serves as a feasible tool for further analysis [55].

This thesis will describe a non-cooperative as well as a cooperative model of a situation of conflict. The first attempts to combine the theory of games with reverse logistics will be analysed next.

2.3 Game theory approaches in reverse logistics

Closely related to the research of this thesis are three papers that focus on the evaluation of different reverse channel structures from a manufacturer's point of view towards an overall profit optimisation. The papers combine the field of research on reverse logistics with game theoretical approaches. That is why, in the following section these papers are presented and their limitations are pointed out.

2.3.1 Closed-loop supply chain models with product manufacturing

Savaskan et al. [73] focus on the manufacturer's problem of choosing an optimal reverse channel structure to supply returns for the re-manufacturing process that are incorporated in the plant to lower manufacturing cost. After the manufacturer distributes the products through a retailer network to the costumer, three different decentralised options of collecting the used products involving the manufacturer (M), a retailer (R) and a third-party (3P) as reverse logistics channel members are investigated. The three reverse channel structures are illustrated in Figure 2.2. The channel members of the structures are modelled as decentralised decision-makers with the manufacturer acting as the Stackelberg leader by taking the first decision on the rate of collection [73].

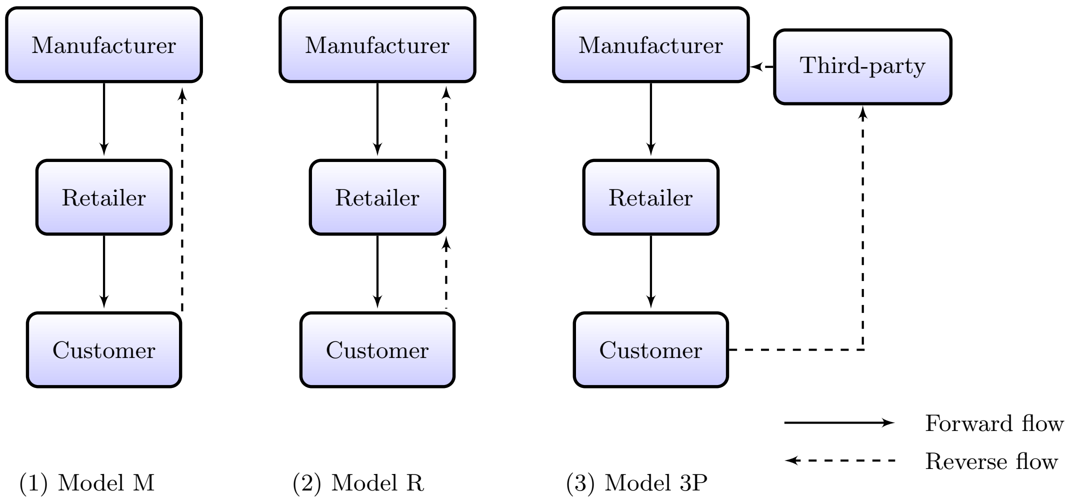


FIGURE 2.2: *The three reverse channel options: Model M, Model R and Model 3P. Source: [73]*

The first option is that the manufacturer collects the used products. Thereby, the manufacturer decides on the return rate as well as wholesale price of the product to maximise the overall profit of this supply chain model [73].

The retailer that sells the manufacturer's products to the customer, additionally collects the returns in the second option. By carrying out the collection effort the retailer decides on the return rate and the retail price per product. The manufacturer has to pay a self-determined transfer price per unit to buy the used product back, since the ownership of the used products lies with the retailer after collection. The manufacturer uses the wholesale price to maximise profits in this channel. With the retailer being the collecting agent, the marginal profitability from a unit increase in demand is higher than through manufacturer collection, because the purchase of a new product also contributes to retrieve the value of its return. Increased profits can be obtained by an investment in the collection of used-products, since scale effects make a larger market size more profitable or by an increase in demand through reduced retail prices if savings are passed on to the retailer. It is beneficial to the manufacturer to pass on savings to raise product demand and thereby increase profits instead of suffering from the problem of double marginalisation by internalise unit cost savings directly. This model shows that for the manufacturer and the retailer pricing decisions in the forward channel are influenced by the way of sharing cost savings from the reverse channel. The final demand of the products also reflects savings from re-manufacturing [73].

The third option is that the manufacturer subcontracts the collection of returns to a third-party logistics service provider and determines the transfer price that buys back the returns. Therefore, the manufacturer distributes products through the retailer but neither of them takes part in the collection effort. The third-party uses the value of the return rate to maximise profits. The retailer decides on the retail price per product, since the retailer only takes part in the forward channel. According to the first two options the manufacturer tries to maximise overall profits by setting the wholesale price. The transfer price incentives the third-party directly to invest in the collection of returns to increase profits. The manufacturer's profit is at balance when both equally share the savings. However, in this channel the transfer price can be seen as a direct cost to the manufacturer [73].

Savaskan et al. [73] observed from their model that the closer a collecting agent is to the market, the more efficient the collection of used products can be carried out. Hence, the manufacturer's overall channel profit is the greatest in the retailer collection, followed by the manufacturer being the collecting agent and is the lowest with the third-party engaging in the collection. Through decentralised collection the demand can be influenced more directly as in central collection. Savaskan et al. [73] provide the idea of a simple two-part tariff between manufacturer and retailer to further improve the profits of the decentralised retailer collection. Therefore, an increase in the collection rate increases the variable part of the two-part tariff.

2.3.2 The case of competing retailers in reverse channel design

Depending on the product characteristics, the forward channel structure and the industry experience, the set up of the channel for collecting used products varies greatly. That is why, Savaskan et al. [74] carried on with their research, trying to evaluate when a manufacturer would make use of competing retailers for the collection of returns.

Savaskan et al. [74] formulate two decentralised closed-loop supply chain channel structures. In the direct collection structure, the manufacturer distributes products via retailers, but collects returns from the customers. The immediate effect of cost savings through product re-manufacturing results in an increase in demand and a maximisation of the manufacturer's profit. In indirect collection, the manufacturer distributes and collects returns via two competing retailers. In this channel a different set of factors influence profitability. Low retail prices due to retailers competing over the collection activity increase demand and the manufacturer's profit. However, if the products of retailers are highly substitutable, the retailers compete more intensely. That is why, the forward channel faces the problem of double marginalisation to a smaller extend. This favours the direct collection structure from a manufacturer's point of view.

The conclusions of Savaskan et al. [73] are extended to a competitive retailer environment. In direct manufacturer-managed collection, cost savings through re-manufacturing drive investment in collection. Retailers prefer the direct option, since they benefit from lower wholesale prices without having to invest. In indirect retailer-managed collection, on the other hand, the manufacturer does not have to invest. Additionally, the manufacturer benefits from an increase in sales volume as the retailers compete through a reduction of retail prices. The manufacturer prefers the indirect option, due to this interaction and especially if the retailers can make independent pricing decisions while their products are not direct substitutes. However, if substitutability of the products is high, direct collection is preferable again. Both, manufacturer and retailer prefer the indirect channel structure if the proximity to the market has a cost advantage. The manufacturer requires a lower cost advantage in this channel than the retailers. In the reverse channel, buy-back payments introduced to coordinate wholesale prices across retailers of different market sizes benefit the manufacturer [74].

2.3.3 How collection cost structure drives a manufacturer's channel choice

Atasu et al. [3] extend the research of Savaskan et al. [73] by incorporating two components into the collection cost. One component is the reverse logistics cost that is dependent on the volume of returns and thereby reflects the effects of economies and diseconomies of scale. The other component is the investment cost that captures the influence of the return rate analysed over a broader set of parameter values than in Savaskan et al. [73]. Due to these extensions, Atasu et al. [3] state that a manufacturer's reverse channel should be shaped according to an analysis of the collection cost structure. Additionally, the model is extended to differentiable new and re-manufactured products.

Similarly to Savaskan et al. [73], the manufacturer sells products through a uncoordinated decentralised two-echelon supply chain made up of a manufacturer and a retailer in the forward channel. In the reverse channel the collection is carried out by the collecting options of the manufacturer, a retailer or a third-party [3].

By analysing the influence of the newly introduced components, there are different observations for scenarios with economies and diseconomies of scale. Under economies of scale in the volume of collection, a linear increase in revenue can be obtained. Hence, profit is convex increasing with the volume of collection in the reverse channel. An optimal solution, in case that the collecting agent finds collection for any quantity profitable, lies in choosing the largest volume of collection or no collection. If the manufacturer manages collection, benefits from economies of scale encourage the manufacturer to increase sales volumes. Unfortunately, lowering the wholesale price to incentive the retailer to sell more is no powerful mechanism any more. The same effect occurs with the collection being subcontracted to a third-party. Additionally, the manufacturer has to share the profits from the reverse channel with the third-party making it the least profitable channel option. If the retailer manages collection and achieves an optimal return channel profits by collecting as many units as possible, the manufacturer can motivate the retailer to sell more by increasing the transfer price. That is why, in economies of scale the manufacturer yields the highest profits with retailer-managed collection. Under diseconomies of scale in the volume of collection, the profits in the reverse channel increases with a decrease in collection rate. In the retailer-managed option the manufacturer's tool of increasing the transfer price only leads to a moderate increase in sales and thus is less effective since it cannot compensate the manufacturer's reduced margin caused by raising the transfer price. The manufacturer would carry out the collection and not pass them on to either the retailer or the third-party, since the manufacturer receives the full benefit of re-manufacturing cost savings. The investment in collection cost function performs basically in the same way as the economies of scale because the average collection cost decreases as sales volumes increase with a rising collection rate. Nevertheless, a distinction lies in the fact that this scale effect has to be strong enough to make the retailer-managed option preferable over the manufacturer-managed reverse channel. In case the investment cost coefficient becomes too high and thereby the absolute cost of collection is too great, the manufacturer collection is an optimal choice. The third-party reverse channel is always dominated by either retailer or manufacturer as it cannot pass on scale effects [3].

Atasu et al. [3] conclude that scale effects within either reverse logistics cost or investment cost are the key to identify an optimal reverse channel choice for a manufacturer. The manufacturer would prefer the retailer to collect returns in case of economies of scale in reverse logistics cost as well as a sufficient scale effect in the investment cost that incentives the retailer to increase sales volume. With diseconomies of scale and a weak scale effect in the investment cost the manufacturer would choose to manage collections. Finally, the manufacturer would not prefer the third-party-managed option in one of the analysed cases. That is why, the optimal choice depends on the three factors of whether there are economies or diseconomies of scale expressed through a scale economies coefficient as well as the magnitude of reverse logistic cost denoted by a reverse logistics cost coefficient and the investment cost described by an investment cost coefficient.

2.3.4 Closed-loop supply chain models for a high-tech product

Chuang et al. [12] make use of a stochastic news vendor modelling framework to analyse a short life cycle high-tech product facing an uncertain demand instead of focusing on a long life cycle product that has a deterministic demand. Additionally, to overcome the tragedy of the commons dilemma that states that manufacturers choose a low collection rate, since re-manufactured products only sell at a low margin, they introduce a collection rate that is exogenously mandated through take-back legislation.

The conclusions of the research are consistent with Savaskan et al. [73] and Atasu et al. [3] with no difference in an endogenous or exogenous mandated collection rate. However, if there are diseconomies of scale and a legislator imposes a collection rate that lies above the threshold level exogenously, the manufacturer's overall profit declines leading to a decrease in product availability for customers. The threshold quantity in the retailer option is half the quantity in the manufacturer collection. Secondly, asymmetric collection cost structures can turn the favour towards the collecting agent, in this case either manufacturer or retailer, with the lowest collection cost. The collection cost per agent can be asymmetric, due to differences in collection technologies as expertise and networks amongst other factors. If the third-parties' collection cost is significantly low at an optimal order quantity, the manufacturer can choose the third-party as an operating agent [12].

With economies of scale that can be achieved through a drop-off collection method for example, the manufacturer's optimal order quantity is the same for every decentralised collecting agent as the manufacturer's economic trade-off stays the same. In diseconomies of scale that might occur with a pick-up collection method for example, the manufacturer's optimal order quantity is lower with the retailer carrying out the collection. The manufacturer's marginal cost then is higher with the retailer operating the reverse channel. The conclusions on the effect of economies and diseconomies of scale on the manufacturer's profit match the results of Savaskan et al. [73] and Atasu et al. [3] even though they focus on long-life cycle products with deterministic demand [12].

In case where the collecting agent chooses the rate of collection endogenously, the return rates of the manufacturer and the retailer are the same and both agents try to collect as many returns as possible under economies of scale. Nevertheless, under diseconomies of scale the channel choice depends on the profitability of the product. The tragedy of commons dilemma results in the manufacturer choosing no or a very low collection rate within the manufacturer's collecting channel. Nevertheless, there are cases in which the return rate is exogenously mandated through different forms of take-back legislation such as individual or collective manufacturer responsibility, manufacturer- or customer-pay and manufacturer-run or state-run collection for example [12].

2.4 Literature evaluation

The following section highlights the main focus of this project by drawing a conclusion from the literature on the application of game theory on reverse logistics design decisions. It combines the developments in research on closed-loop supply chain and game theory.

Different approaches to deal with problems arising from the integration of reverse logistics to create closed-loop supply chains have been applied. Game theory seems to be a promising approach to analyse the various interdependences, since one of the major differences towards traditional supply chains is the amount of individual decision-makers dealing with reverse flows. Especially as game theory does not only provide solution concepts for non-cooperative conflict situations but also allows to incorporate a cooperative point of view. Specifically the articles of Savaskan et al. [73], Savaskan et al. [74], Atasu et al. [3] and Chuang et al. [12] develop the idea of this application further.

Savaskan et al. [73] model a single-period game where the different channel members are described through a single manufacturer, a single retailer and a single third-party collector monopoly with the manufacturer having enough market power to act as a Stackelberg leader. The rate of collection is endogenous mandated by the channel members. They assume perfect substitution between new and re-manufactured long-life cycle products. The deterministic demand for these products decreases linearly with price. Additionally, the collection cost structure is modelled independent of the collecting agent and does not exhibit economies of scale as it increases linear. By carrying on with their research to investigate the model in an environment of competing retailers, Savaskan et al. [74] break up the assumption of a retailer's monopoly. Besides this extension, the assumptions stay the same. The results attained in the earlier article that closeness to the market is crucial to find an optimal reverse channel structure are supported by the new conclusions [2].

Atasu et al. [3] extend the work of Savaskan et al. [73] by modelling a volume-dependent reverse logistics cost under different operating environments. Therefore, the effects of economies and diseconomies of scale are taken into account. The extension favours the collection through the retailer's channel, as scale effects can be passed on easily to the members of this channel. Besides, similar results are attained by introducing differentiable products into the model. Nevertheless, all three channel members still facing the same collection cost structure even though scales are likely to tip in favour of the party with the lowest reverse logistic cost.

Even though Chuang et al. [12] focus on short-life cycle products with an exogenous mandated collection rate their results are consistent with Savaskan et al. [73] and Atasu et al. [3]. Additionally, the influence of asymmetric reverse logistic costs due to differences per agent are briefly discussed in the course of the paper.

All articles develop an analytical model to address the question of who should carry out the collection of returned products. Even though most of the assumptions taken from the research of Savaskan et al. [73] are validated and the following conclusions show consistency, the important impact of reverse logistic cost individual to each of the collecting agent has not been analysed yet. That is why, this thesis will focus on the impact of reverse logistics cost. The interactions between the players will be simulated to generate cost that are as realistic as possible. Additionally, the interrelations of the three independent decision-makers of manufacturer, retailer and third-party that incorporate different reverse logistics costs will be analysed in a non-cooperative and a cooperative model of the game. Reasons for cooperation can be either economic or laws imposed by the legislative.

CHAPTER 3

Modelling reverse logistics as a game

Contents

A description of a game must include the elements of a set of individual decision-makers, the possible actions available to each actor and the rules prescribing the game [54]. The finite set of players in this model consists of the manufacturer (M), the retailer (R) and the third-party logistics service provider (3P). The characteristics of each player will be described next, resembling their network structures for collection and focusing on their maximisation intentions. The game will then be solved playing for the level of collection rate. The manufacturer acts as the Stackelberg leader having the greatest market power compared to the other two collecting agents. Therefore, the manufacturer's optimal collection rate is fulfilled first followed by the retailer and the third-party as part of the rules of the game.

The game itself has to be characterised to apply the techniques of game theory to the competition between the three independent decision-makers. The game classifies as a three-person game, since it is made up of three different players. The possibility of collaboration between two players increases as soon as a third player is added. That is why, the main focus of n-person game theory lies in the dynamics of formation or disintegration of coalitions as well as the corresponding payoffs [95]. Even though this game is modelled as a situation of conflict in competing over the number of used products available for collection first, the players might benefit from a collaboration as their interests are not of total conflict. Taking the cost structure of the payoff function into account, the opportunity to lower product prices and increase demand is available through collecting as many returns as possible. Therefore, increasing the return rate in the non-cooperative or cooperative version of the game can affect the overall profit of a channel member in a positive way.

The question that will be addressed in this project is what reverse channel structure is best for the collection of used products. This question will be answered from the decentralised view of every player in the game. Besides testing the stability, different scenarios will be covered to evaluate the results in various environments. Additionally, special attention will be given to the manufacturer's perspective as the manufacturer will most likely be confronted with the responsibility for the recovery processes by legislation. If the manufacturer has to maximise the amount of collection, which reverse channel should be chosen?

There are several assumptions that need to be made to enable the analysis of the game. These assumptions will be explained in Section 3.1 to describe the market environment in which the game takes place. The profit per returned unit is highly dependent on the cost of collection. This cost of collection varies significantly among the players. The reverse logistics network of each player is responsible for the collection of returns from customers and the transport to recovery facilities. The network choice is the most crucial component of closed-loop supply chain design, since transport cost influences the economic viability of product recovery to the largest extend. The number of re-manufactured products and the correlating profits are determined by the quantity of collected products [1, 30]. That is why, the collection structure of each player will be detailed in Section 3.2. In Section 3.3 the corresponding payoff functions will be introduced and the influences of the cost of collection will be pointed out. In conclusion of the methodology, the game is solved for a benchmark scenario in Section 3.4 to illustrate the competitive situation of a non-cooperative game.

3.1 Market mechanisms

In this section the assumptions that are applied to characterise the market mechanisms are presented. These mechanisms help to focus on the main conclusions of the study. As the primary goal of this project is to investigate the consequences of different reverse channel structures, the following specifications will help to create an environment for possible investigations.

The game will include three different reverse channel structures modelled by three different players. Each player has a specific payoff function. Profit is derived from revenue minus cost and thus both components have to be defined. Therefore, Assumption 1 and Assumption 2 specify the manner in which revenue can be earned, while Assumption 3 and 4 deal with the cost arising from the collection effort. General rules on how the search for an optimal collection rate proceeds between the three players are introduced by Assumption 5 and Assumption 6. Finally, the modelling of reverse logistics as a game will result in the competition for an optimal collection rate between the three different players.

ASSUMPTION 1:
The demand of the price per unit p is given by $D(p) = \phi - \beta p$, where ϕ and β are positive parameters and $\phi > \beta c_m$ with c_m describing the cost of manufacturing a product [73].

With this assumption the demand is modelled as a linear downward sloping function. This means that profit is achieved by the price per product times the amount of products sold. However, a saturation of demand occurs after a certain amount of time that products have been released.

ASSUMPTION 2:
Re-manufacturing used products directly saves manufacturing costs, since manufacturing a new

product c_m is more expensive than recovering a returned good c_r, i.e. $c_m > c_r$ [74].

The manufacturer has incorporated a recovery opportunity into the existing manufacturing process to be able to produce a new product from recovered material or return directly. A higher product return rate is strictly preferred over a lower rate with this assumption. Not only selling new products, but also selling used products back to the manufacturer for recovery can be a source of income for a player. Uncertainty lies in the usability of returned products, due to different application possibilities. This uncertainty in the quality of the return results in a reduction of incentives to invest in collection as the benefit from the investment might be lower [73].

Assumption 3:
The collection rate of returns from customers is described by the reverse channel performance Θ. This performance denotes a fraction of the products re-manufactured from returns, i.e. $0 \leq \Theta \leq 1$. The investment in collection activities, denoted by I, describes the product collection effort as a function of Θ [74].

The collecting agents' promotional activities stimulate customers to return their products for re-manufacturing resulting in the response Θ. There are no direct financial offers given to the customers, thus the model focuses on the channel behaviour of the agents, while the customers only react to new product prices. Reverse logistics as well as the investment in collection leads to the cost of collection. The diminishing return on the investment is represented by the cost structure $\Theta = \sqrt{I/R}$, where R is a scaling parameter. Consequently, the average unit cost of manufacturing is $c = c_m(1 - \Theta) + c_r\Theta$. If the unit cost saving from recovery is described by Δ with $\Delta = c_m - c_r$. These assumptions lead to an average unit cost given by $c_m - \Theta\Delta$ [73, 74].

Assumption 4:
The scaling parameter R is sufficiently large to satisfy the condition that $\Theta < 1$ [73].

Re-manufacturing all products from returns is not economically viable. Therefore, this assumption ensures that the cost of re-manufacturing are high enough. The economic law of diminishing returns on investment states that if one input in the manufacturing of a unit is increased but all other inputs are held at a fixed level, a point where additions to the input result in smaller or even diminishing increase in output will be reached eventually. A classical example of this law is that a farmer finds a specified number of workers yield the maximum output per labourer on his land. The combination of land and work is less efficient if he hires more labourers, since the proportional increase of overall output is smaller than the expansion of workforce. Therefore, the output per labourer would decline. Unless the technique of manufacturing is changed additionally, this law holds true for any manufacturing process [19].

The company has to make expenses on advertising to make customers aware of the possibility of recycling end-of life products. Even if the collection is carried out by one of the three collecting agents, the connection between customer and company has to be arranged. Advertisement for the collection of used products can be seen as a starting point of the entire collection process. Therefore, according to Savaskan et al. [73], the diminishing return on investment I is characterised by

$$I = R\,\Theta^2. \tag{3.1}$$

The scaling parameter R has to be large enough to prevent the manufacturing of all products from returns as stated in Assumption 3, R has to satisfy

$$4\,R > [(\phi - \beta c_m) + \beta(\Delta - C_D)](\Delta - C_D), \tag{3.2}$$

where C_D is the reverse logistics cost per unit and the right-hand side of the condition resembles the maximum savings contribution of re-manufacturing with $\Theta = 1$ describing the recovery of all products. In addition, each collecting agents has a different rate of collection Θ to compete over and a different scaling parameter R as the proximity from the agent to the customer differs.

Response functions in advertising and operation literature use similar trade-offs as in the remanufacturing context. In advertising, response models for the retention of customers and product awareness are applied to describe the relationship amongst the components of the model. An optimal advertising strategy is determined by a market situation in which two companies compete over customers with the help of advertisement. Thereby, advertising expenditures for closed-loop strategies are proportional to spendings in open-loop advertising where players cannot observe the play of competitors and the square of the opponent's actual market share [33]. With the introduction of a new product, product awareness needs to arise from potential customers. A companies' optimal advertising and pricing strategy is determined by the signal the product price sends to the customers defined by the level of investment [96]. The operation models use specific investment functions describing the interrelations to investigate on possibilities for the improvement of processes and lot sizing by investing in reduction of set-up cost and quality improvement. There is a connection between quality and lot sizing as processes producing large amounts can be faulty each time another unit is produced. An extra cost for re-work of defective units has to be incurred, resulting in an incentive to produce smaller lots with higher quality units. This is leading to a specific form of investment cost function [66]. In dynamic process improvement, the immediate return on investment is the expected net value of reducing operating costs if no additional investments are made. The last chance policy determines an optimal amount to invest for maximising the return on investment. Therefore, it is optimal to invest in process improvement if a positive amount is invested with the last chance policy [25].

ASSUMPTION 5:
All agents have the same access to information while trying to optimise their objective function [73].

Information asymmetry might result in inefficiencies and risk that can be controlled with the help of this assumption. Therefore, it creates an equal information base for every agent to optimise its performance. None of the players has an advantage over the competitors when it comes to information.

ASSUMPTION 6:
The manufacturer has enough channel power to act as a Stackelberg leader over the retailer and the third-party [73].

Even though it is preferable to collect a high rate of returns, the rate cannot exceed its upper bound of 100%, due to the third assumption. Therefore, the manufacturer gets to decide first how many returns to collect leaving the retailer and third party to compete.

Taking these assumptions into consideration for the general market model, different scenarios can be modelled from this foundation. Nevertheless, changing these assumptions may lead to completely different results.

3.2 Reverse logistics cost function

The cost of collection is a function composed of reverse logistics cost and investment cost. The investment cost I is the incentive that allows effective collection of returns from customers. The reverse logistics cost C_D is the cost of transporting used products to the manufacturer for recovery purposes. A function of the return rate of used products characterises the total cost of collection $C(\Theta)$. It is given by $C(\Theta) = C_D(\Theta)D(p) + I$. The cost of investment I has been described in the assumptions for the market mechanisms. Especially the cost components of the reverse logistics are crucial to this project. Therefore, the reverse logistics cost functions are defined in detail for each collecting agent in the following sections [73].

Reverse logistics networks facilitate the collection of returns from customers in the market area and the transport to the manufacturer's recovery facility. For each of the three different collecting agents, there are three different designs of reverse logistics network. A careful design and control of the collection network is crucial in closed-loop supply chains, since transport cost influences the economic viability of re-manufacturing to a large extend [30]. That is why, different methods of transport, economies of scale in transport and a methodology to approximate logistics cost will be discussed next. Finally, the reverse logistics cost function per player is derived.

3.2.1 Different methods of transport

Two methods of transport, one for direct shipping and one for multiple stop collection, are examined for one agent collecting units via trucks from many customers. The objective of economic order quantity (EOQ) models is to minimise the sum of inventory carrying and ordering cost. Key factors in these formulas are customer supply and density, transport cost per distance as well as value of items and charges for inventory carrying. The analysis of the two collection methods will illustrate the differences in the reverse logistics network design of the three agents [10].

In direct shipping, units are shipped directly from one customer to the manufacturer's recovery facility. Evaluating the trade-offs between transport and inventory cost per unit, a function for shipment size specifies an optimal lot size and thus defines the minimum transport cost per unit. For every collection trip to a single customer, the cost components of loading, transporting and unloading determine the transport cost per unit. The decrease in transport cost per unit with shipment size reflects the economies of scale in shipping. Let

K_L be the cost per stop at a customer (loading cost),
K_U be the cost to dispatch a vehicle and stop at the service facility (unloading cost),
T be the transport cost per distance per unit,
ℓ be the distance between customer and service facility, and
S be the lot size.

The direct shipping transport cost per unit T_D is thus calculated by

$$T_D = \frac{K_L + 2\,T\,\ell + K_U}{S}\ . \tag{3.3}$$

The inventory cost in reverse logistics is represented by a holding cost. That is why, the holding cost per unit depends on time spend waiting at the customer, lead time of the recovery and the time spend before further treatments of the return at the recovery facility. The average time to form a shipment of size S is determined by the supply of returns, since products reach their end of life independent from scheduled pick up times. Therefore, each unit that is part of a shipment waits at average half this time before being shipped. If the recovery of used products is carried out at a constant rate, the average time to empty a truck is also dependent on the lot size per return rate. After arrival at the recovery facility, a unit waits half this time on average again. Let

r be the holding cost rate,
Θ be the return rate of used products, and
U be the transit time.

The direct shipping holding cost per unit H_D is then calculated as

$$H_D = r\left(\frac{S}{2\Theta} + U + \frac{S}{2\Theta}\right). \tag{3.4}$$

Large volume shipments reduce transport cost but raise inventory cost, as inventory cost per unit increases linearly with shipment size. An optimal lot size of a shipment S^* is given by minimising the first-order conditions of the total cost $(T_D + H_D)$. Therefore, let

V	be the vehicle capacity, and
S_{max}	be the accumulation capacity of a collection point.

An optimal size of a shipment S^* in direct shipment is then given by

$$S^* = \min\left(\sqrt{(K_L + g\,T\,\ell + K_U)\left(\frac{\Theta}{h}\right)}, V, S_{max}\right). \tag{3.5}$$

An optimal shipment size S^* does not necessarily need to be a full truck load. In reality, linear assumptions on shipping cost only hold true if $S = V$. With $S < V$ economies of scale have to be considered [8, 10]. According to Beullens et al. [8], in reverse logistics the value of a unit largely exceeds the transport cost, thus in many cases $S^* = S_{max}$. Vehicles that are able to access the road infrastructures leading directly to customers cannot handle the large volumes of returns required to make the collection profitable. This issue limits the success of recycling, especially when transport is carried out via direct shipments.

The cost of direct shipping increases when a fixed number of returns is collected among many customers with smaller than the EOQ per vehicle. Lower volumes per collection point per trip are possible, if a vehicle makes multiple stops per collection. Therefore, the market area is divided up into subregions forming individual service areas. Each service region defines a unique group of customers providing the basis for defining an optimal quantity of units per truck load. A tour with multiple stops per collection is also called a milk-run and combines the three stages of line-haul, local and back-haul transport. The line-haul defines the unloaded travel from the service facility to the nearest customer. In this case, a one-way haul beginning with an empty truck is described. The travel between the first and the last customer stop on the tour is part of the local transport. The back-haul is the loaded travel from the last customer stop back to the service facility that forms the end of a tour. Line-haul and back-haul equations similar to direct shipping are applied and local transport cost is added to analyse the transport cost. The results of Burns et al. [10] on the investigation in the EOQ of milk-runs reveal that trucks should always be despatched full. Therefore, the average distance per stop is a function that is described by the Euclidean distance $\ell(x)$ between the service facility and the centre of the service region x served by a fully loaded vehicle for line-haul and back-haul. Additionally, the point density in x determines the local collection distance $\rho(x)$. This function, that is depend on the location, slowly varies in x within the overall market area A containing a total of n collection points. It follows that the average milk-run transport cost per unit M as a function of x may be calculated as

$$M(x) = \frac{K_L}{S} + \frac{K_U}{V} + T\left(\frac{2\,\ell(x)}{V} + \frac{\sqrt{1/3}}{S\,\sqrt{\rho(x)}}\right). \tag{3.6}$$

Integration over the market area A gives M_A. Therefore, the average milk-run transport cost per unit M_A becomes

$$M_A = \frac{K_L}{S} + \frac{K_U}{V} + T\left(\frac{2\,I_1}{V} + \frac{\sqrt{1/3}\,I_2}{S}\right), \tag{3.7}$$

with $I_1 = \frac{1}{n}\int_A \rho(x)\ell(x)\mathrm{d}x$ and $I_2 = \frac{1}{n}\int_A \rho(x)^{\frac{1}{2}}\mathrm{d}x$.

The holding time in a milk-run is extended by the transit time of local transport U_L. The units collected at the first stop do not incur transit time, while at the last stop of the route they get full transit time. This results in an average of $U_L/2$ for all units. The additional transit time of local transport U_L is appended to the transit time U of direct shipment in equation (3.4). Assuming that time is related to distance for integration over the market area, the average milk-run holding cost per unit H_M is given by

$$H_M = r\left(\frac{S}{\Theta} + b\,I_1 + \frac{\sqrt{1/3}\,l\,I_2}{S}\right), \tag{3.8}$$

where b is the constant expressing the average speed on the back-haul trip and l on the local collection tour.

An optimal lot size S^* in milk-run collection is calculated by minimising the first-order conditions of the total cost $(M + H_M)$ leading to

$$S^* = \min\left(\sqrt{\left(K_L + \sqrt{1/3}\,T\,I_2\right)\left(\frac{\Theta}{h}\right)}, V, S_{max}\right). \tag{3.9}$$

The average total cost per unit in milk-run collection is a function of the point density ρ that is continuous and decreases monotonously [8].

Multiple stops per tour often reduce the cost of collection compared to direct shipping. Direct shipping is only cheaper for lower ρ, h, K_U, ℓ and the better the correlation between vehicle and accumulation capacity of the collection point. Therefore, balancing between transport cost and service aspects due to the lower value of the returns becomes important [8, 10].

The three collecting agents have different network designs. The manufacturer can only choose between direct shipping and milk-run transport. Due to generally lower transport cost with multi-stop vehicles the manufacturer only uses milk-runs in the collection channel. The retailer as well as the third party can combine the two transport modes to exhaust the advantages of each option. The retail stores of the retailer can be used as collection hubs to combine the returns collected by milk-run tours in the service regions to direct shipments towards the manufacturer's recovery facility. In contrast to the third-party, the retailer is bound to the use of already installed stores as consolidation points. The third-party is relatively free in choosing consolidation points due to the possibility of combination of collection with other clients. That is why, the third-party can choose an optimal size of a collection area and thereby reduce the transport cost of collection. The reverse logistics per collecting agent is described in detail next.

The number of re-manufactured products is determined by the quantity of collected returns. This quantity is expressed by the collection rate Θ that lies in the interval $[0, 1]$. As re-manufacturing leads to a direct saving in manufacturing cost in this model, it is favourable to collect as many returns as possible taking the costs related to operating a reverse channel into consideration.

Savaskan et al. [73] use an acquisition cost per unit C_A to describe the reverse logistics cost of each channel. This acquisition cost can be seen as a fixed payment for each returned product. In this case, no consolidation of goods takes place and thus the reverse logistics cost can be computed with equation (3.3) as in direct shipments, assuming the acquisition cost to be an average cost per transport per unit. As the reverse logistics cost is dependent on the return rate, $C_A(\Theta)$ depicts an increasing linear function. The only difference between the three collecting agents lies in the level of acquisition cost per unit. With decreasing distance to the customer the acquisition cost per unit diminishes as the transport effort is reduced. Therefore, the relation between the acquisition cost of the different collection channels M, R and 3P, represented by the superscripts before the cost, is resembled by ${}^{M}C_A > {}^{R}C_A > {}^{3P}C_A$.

3.2.2 Economies of scale in transport

Direct shipment is not always the cheapest mode of transport. Additionally, the acquisition cost does not resemble any economies of scale. Baumgartner et al. [7] point out that ignoring economies of scale and transport frequencies can result in significantly higher cost. The most common scale economy effects in supply chain modelling are scale economies with transport distance and scale economies with transport quantity. The cost-capacity factor will be taken into account at a later stage of the reverse logistics cost formulation.

The correlation between transport cost and distance is defined to consider the effect of distance on cost in a realistic way. This correlation is expressed as a cost function per distance per transported unit. The cost function is convex and monotone increasing with distance. There is a non-direct proportional increase in rates with distance, due to the possibility of distributing the transport cost over greater distance. In an analysis of several cost generators in forwarding by Forkenbrock [32] the unit cost shows a decreasing tendency over the total travel length. Based on the data of the forwarder in the analysis of Forkenbrock [32], Vieira et al. [90] established a cost variation model that represents the economies of scales with distance. The result is a correctional factor Ψ_ℓ that can be applied to cost per distance to condition the transport cost. Therefore, let

ℓ_{max}	be the maximum distance between two entities of a supply chain,
ℓ	be the distance between the facilities that are analysed,
α_ℓ, η_ℓ	be the parameters to fit the real data used, and
z	be the parameter that describes non-considerable variations of the cost-ratio.

The correction factor for distance can then be expressed dimensionless by

$$\Psi_\ell = \eta_\ell \, e^{\alpha_\ell \left(\frac{\ell}{\ell_{max}} \right)} + z \; . \tag{3.10}$$

The variations of the correctional factor Ψ_ℓ, with the data represented by Forkenbrock [32] where the maximum distance is $\ell_{max} = 200$ km, $z = 0.8$, $\eta_\ell = 0.214$ and $\alpha_\ell = -2.8$ are illustrated in Figure 3.1 [90].

The plot in Figure 3.1 shows that the smaller the difference of the distance between two facilities and the maximum distance, the smaller the correctional factor becomes. Therefore, the correctional factor strives towards z. This results in a convex function for the variations of the correctional factor Ψ_ℓ with distance. The distribution of cost over distance will be applied to the reverse logistics cost in this model when comparing scenarios with different market area sizes.

The size of a shipment is denoted by V, thus V_{max} denotes a full truck load. For a defined section of the transport distance, units are collected with identical vehicles that are capable of carrying full truck loads. The concept of an unit can be redefined if different types of returns are moved through a collection system as an unit can be either of volume or of weight and V_{max} can be the vehicle's volume or weight capacity. This makes the maximum freight volume or weight of a vehicle independent of the mixture of return types forming the load. Whenever a multiple of V_{max} is reached or exceeded by sending very large shipments, a new truck is dispatched resulting in a jump of transportation cost. An operating truck is insensitive to the shipments it carries, thus the steps of the transport cost curve are rather flat. Nevertheless, sub-additivity ensures

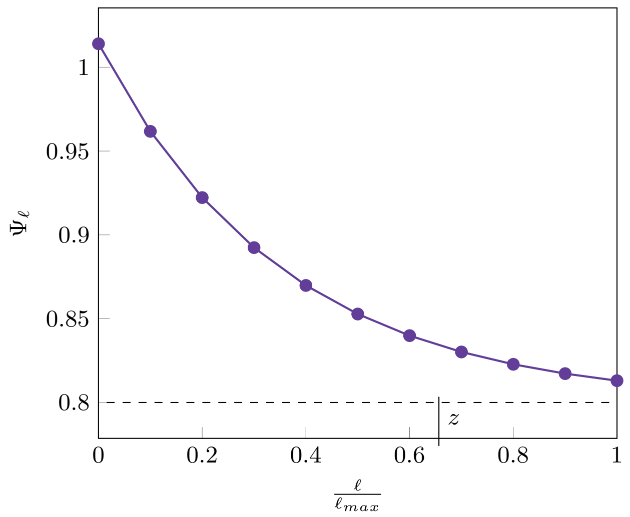


FIGURE 3.1: *A plot of the variation of the correctional factor Ψ_ℓ with distance. Source: [90]*

that sending shipments in parts does not reduce transport cost. The largest size of vehicles that can still accommodate the local roads should be used as the minimum transport cost that is a decreasing function of V_{max} [13].

The existence of economies of scale with quantities is emphasised in Lapierre et al. [53] by increasing the shipment size to a maximum. This cost variation is defined as a non-linear function of volume due to discounts with increasing quantities. Taking a maximum limit into account, the transport frequencies are considered indirectly. The minimisation of shipments results in the lowest cost achievable. The model formulation will condition the transport frequencies to find the lowest cost. Vieira et al. [90] use the analysis in Lapierre et al. [53] to create a correctional factor Ψ_q for transport quantities. Let

q	be the quantity of the transported product,
q_{max}	be the maximum truck capacity for the product, and
α_q, η_q	be the parameters to fit the real data used.

The dimensionless correction factor for quantity may then be expressed as

$$\Psi_q = \eta_q + \alpha_q \ln\left(\frac{q}{q_{max}}\right). \tag{3.11}$$

Using the case study data of Lapierre et al. [53] with $q_{max} = 3\,000$ units, $\eta_q = 0.2815$ and $\alpha_q = 0.134$, the variation of the correctional factor Ψ_q with the transported quantities is displayed in Figure 3.2. As the difference between maximum truck capacity and transported volume diminishes, the correctional factor with quantities reduces to a minimum until reaching α_q.

3.2.3 Continuous approximation methodology

In this section, the reverse logistics cost function per collecting agent will be modelled to resemble the coherences introduced. Therefore, the continuous approximation methodology, proposed by Newell and further developed by Daganzo [13], will be applied to describe the components of reverse logistics cost and generate non-linearity through economies of scale. The continuous approximation approach is a fairly simple but effective alternative in logistics network design

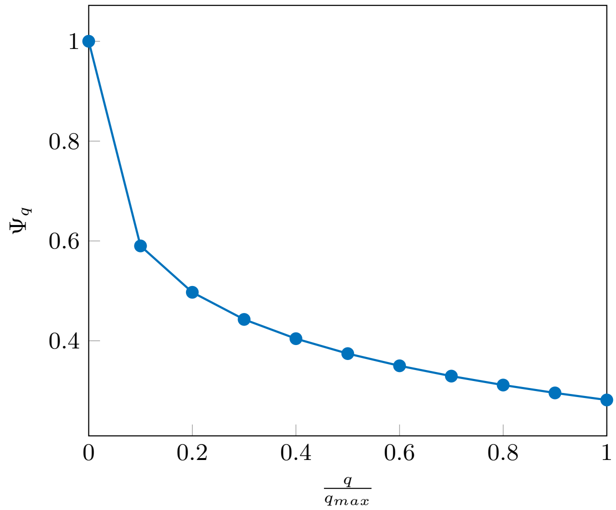


FIGURE 3.2: *A plot of the variation of the correctional factor Ψ_q with quantities. Source: [90]*

to receive general insights into the economics of logistics systems and the preferability of each strategy under different design options. The impact of critical parameters can be revealed based on analytical arguments to draw conclusions on the behaviour of real-world systems.

The continuous approximation methodology provides a way to derive approximate logistics cost per unit. The representation of demand by a continuous density function over the service area instead of a concentration of demand in specific locations is the key element of this approach. In the case of reverse logistics, the demand of products is converted into the function for the supply of returns. If the density is constant or only slowly varying across the service area, logistic cost can be approximated by appropriately chosen averages that can be expressed as simple functions with a limited number of parameters. The second assumption, namely that the facility that is closest would always be chosen to make the collection effort, leads to facilities with equal-sized non-overlapping service areas that serve the market area. Fixed costs associated with operating facilities in the service area as well as variable cost for coordination and transport activities are carefully assessed. The continuous approximation approach estimates the cost accurately and the decomposition principle holds true, if the sum of costs over non-overlapping regions can express the total cost and these cost components are only dependent on the decisions made in their regions. By integrating over the whole service area, the total logistics cost are obtained and divided by the total number of units to find an average cost per unit [1, 13, 28, 30, 93].

A comparison of alternative modelling approaches over a numerical example was executed by Fleischmann et al. [30]. The comparison reveals that the models show similar results in logistics cost. The example contrasts the results of deterministic mixed integer linear programming (MILP) approaches, stochastic mixed integer linear modelling approaches and the continuous approximation methodology. Stochastic MILPs incorporate the aspect of uncertainty explicitly, while deterministic MILPs address uncertainty with the evaluation of different scenarios. The best solution obtained in a deterministic MILP and stochastic MILP only differs within a few percentages. Nevertheless, both MILP approaches have limited capabilities for sensitivity analyses and do not make the interrelations between different parameters explicit. The continuous approximation, on the other hand, is an approach that effectively reveals the interrelations in comparison to the theoretically more powerful but computationally more demanding MILP approaches. Therefore, the continuous approximation approach is chosen to model the reverse logistics cost in this thesis.

In reverse logistics it can be assumed that the supply of returns does not change rapidly. Whether products are returned can be influenced to a large extend by the promotional activities of the collecting agent. The density function of returns can be described as $\rho = \varphi\,\Theta$, with $\varphi \in [0, 1]$ describing the density constant over the market area. The analysis of the reverse logistics cost must include the fixed cost associated with operating a collection facility as well as the variable cost related to transport and coordination activities. Depending on the mode of transport being used, this transport cost component can have different characteristics [1].

Analysing the fixed cost component first, each service facility operates in a circular area with a radius that defines the magnitude of this area. The density of returns inside the facilities' service area reflects the total number of collected used products. An annualised fixed cost represents the necessary expenditures to operate a service facility [1, 30]. This cost component does not vary with volume, because the service facility needs to be able to cover seasonal variations as well as temporary capacity peaks. However, the fixed cost of operation can vary between different sizes of service areas. The smaller the service area, the lower the operational requirements towards the facility. Hence, less cost is associated with operating the service facility for a smaller area than a larger area [34].

Over the service area A the annualised fixed cost of operating a service facility F is denoted as the annualised fixed cost per service facility per unit f given by

$$f = \frac{F}{A\,\rho}. \tag{3.12}$$

In this model, f differs per player since every player operates in a service area of a different size. The manufacturer only uses one service facility that is the manufacturing plant. Nevertheless, the coordination and handling of the returned products requires additional expenditures. Therefore, part of the operational cost of the plant is assigned to the reverse logistics function of the manufacturer. For the retailer every retail store can be used as a service facility for collecting returns. The cost that occurs while carrying out the collection effort in a part of the retail store is associated with the retailer's reverse logistics cost. In general, this cost is assigned to smaller service areas as the market area is divided up between the retail stores. The third-party can make use of synergy effects with other clients but still needs to be able to service the entire market area of the manufacturer and thus takes the fixed cost of operating terminals into account when calculating the reverse logistics cost per unit. The cost of serving the whole market area is generally higher than the cost of serving only part of it.

The variable cost component consisting of transport and coordination between the customer, the service facilities and the manufacturer is analysed secondly. The transport cost per player is dependent on the service areas of the facilities involved in the collection process. In the immediate vicinity of customers only smaller vehicles such as delivery vans can be used due to smaller roads, whereas easier access and consolidation at the service facilities enables the use of larger trucks to carry returns to the manufacturer's recovery facility. The combination of the benefits of direct shipment and multiple-stop transport results in a minimum of transport cost [1, 30].

Due to the limited accessibility of customer collection points, milk-runs are incorporated to reach customers. The line-haul distance between the service facility and the start and end of a pick-up tour and the sum of the distances between two consecutive points of collection are assessed next.

According to Newell et al. [64], the service area is divided into ring-radial zones with nearly rectangular pick-up areas where customers are located. A single vehicle route is optimised starting and ending at the centre of the service area where the facility is located. The pick-up areas near the centre are more wedge-shaped, whereas on the edge of the area the pick-up areas are approximately rectangular shaped. The customers are assumed to lie on a fine rectangular grid of roads. A ring-radial grid may be used to access the collection areas, but the tours that lead directly to the customers are mostly on the fine rectangular grid. A possibility to depict the zoning of a service facilities' area is displayed in Figure 3.3.

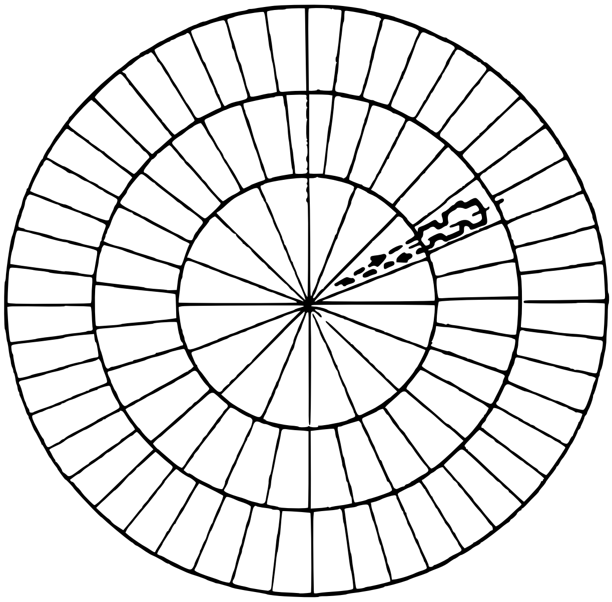


FIGURE 3.3: *A possible zoning of a facilities' service area. Source: [64]*

An example of a tour through one of the approximately rectangular shaped pick-up zones is illustrated in Figure 3.4. Assuming full truck load per collecting vehicle in this zone, the number of customers O in this pick-up zone of length L and width $2w$ is approximately

$$O = 2wL\rho. \tag{3.13}$$

The shortest closed tour connecting uniformly and independently distributed customers O in a pick-up area A is described by the average tour distance d given by $d \cong k\sqrt{O\ A}$ with constant k being set to the value 0.75 based on simulation experiments. This holds true for a distance given in the Euclidean and in the square grid metric. In this project the square grid metric is used as a more realistic approach to find the shortest connection, since streets routes have to pass the buildings in an area and cannot go straight to the destination [9, 14].

Taking the dimensions of a rectangular shaped pick-up area into consideration, as shown in Figure 3.4, the length of a tour as a function of w is estimated by dividing the rectangle into two parts of width w. On the way out of the pick-up zone, the customers are visited in increasing order of the coordinate x along the length L and on the way back to the facility in decreasing order of the coordinate x [64].

In a ring-radial road network as shown in Figure 3.3 the zones would be elongated to the radial direction, so that the total tour length can be divided up into the sum of the longitudinal (radial) and the transverse (ring) parts. Figure 3.5 gives a closer look at a strip of width w with an infinitely length and randomly but uniformly scattered points with point density ρ per area.

A path connects all points that resemble customers in the area. The expected total length of the tour is given by the number of points multiplied with the distance between two consecutive points. If X denotes the random distance between two consecutive points along the width w,

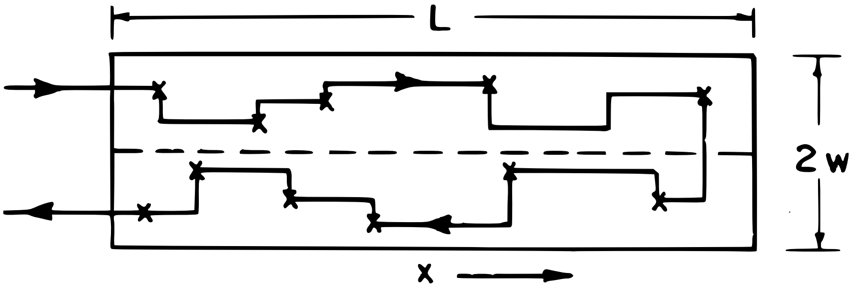


FIGURE 3.4: *An example of a tour in a pick-up zone. Source: [64]*

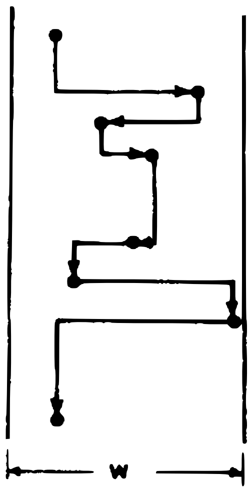


FIGURE 3.5: *A possible strip with width w. Source: [64]*

then the probability that the points are at least at a distance x apart is given by

$$Pr\{X > x\} = \left(1 - \frac{x}{w}\right)^2 \text{ with } 0 \leq x \leq w \; , \tag{3.14}$$

because X has the same distribution as the distance of two random points on a line segment. Integrating over the interval $[0, w]$ leads to $w/3$ that denotes the average transverse tour distance. Along an indefinite length of the strip, Y describes the random distance between two consecutive points with the probability that these points are at least y apart given by

$$Pr\{Y > y\} = e^{-w\rho y} \text{ with } y \geq 0 \; . \tag{3.15}$$

A Poisson counting process with rate $w\rho$ is applied, because the positions of these points lie along the side of the strip and are completely random. The longitudinal tour distance that is needed to traverse the pick-up zone is described by $w\rho^{-1}$ after integrating over the interval $[0, w]$ [72]. Hence, the average local tour distance per customer d_L of the rectangle in Figure 3.4 is denoted by the sum of the average transverse and longitudinal tour distance for the square grid metric calculated by

$$d_L = \frac{w}{3} + \frac{1}{w\rho} \; . \tag{3.16}$$

With a large w the transverse distance becomes longer, but the the longitudinal distance per customer becomes shorter as the width of the rectangle increases. The right balance can be found by identifying an optimal width w^* that minimises the average local tour distance per customer. For the optimal width $w^* = \sqrt{3/\rho}$, equation (3.16) gives a dimensionless local travel distance per point as

$$d_L = \sqrt{\frac{1}{3\rho}} \,. \tag{3.17}$$

With this information, the average local tour distance per customer d_L that is independent of the line-haul distance together with T, the transport cost of a vehicle, forms the vehicle routing cost. According to Daganzo [14] and Newell et al. [64] the vehicle routing cost per unit M_R is calculated as

$$M_R = T\sqrt{\frac{1}{3\rho}} \,. \tag{3.18}$$

The distance between the service facility and the start and end point of a pick-up tour is dependent on the size of the area the facility serves. The average distance ℓ to the service facility can generally be obtained by integration over a service area with $2\rho\pi x$ customers that are at a distance x from the service facility. Therefore, the average distance ℓ is calculated as

$$\ell = \frac{1}{\rho\pi r^2}\int_0^r x(2\rho\pi x)\mathrm{d}x = \frac{2}{3}r \,. \tag{3.19}$$

The average distance for line-haul and back-haul together with the transport cost of a vehicle T per maximum truck capacity V generates the second part of the multiple-stop transport. The total milk-run transport cost between customer and service facility per unit M_C results from the sum of this part and the vehicle routing cost and is given by

$$M_C = \frac{4}{3\sqrt{\pi}}\sqrt{A}\,\frac{T}{V}\,\rho + T\sqrt{\frac{1}{3\rho}} \,. \tag{3.20}$$

Direct shipments from the service facility to the manufacturer are introduced for the retailer and the third-party, due to the consolidation effects of the service facilities. The line-haul cost is obtained in the same way as the second part of the milk-run transport cost. However, the easier accessibility of the service facilities allows the introduction of vehicles with higher transport capacities and correspondingly lower mileage cost. Therefore, the cost for direct shipments between service facility and manufacturer per unit T_C, assuming full truck loads, becomes

$$T_C = 2\,\ell\,\frac{t}{v}\,\rho \,. \tag{3.21}$$

3.2.4 Reverse logistics cost per player

With the help of equations (3.12), (3.20) and (3.21), the variable component of the reverse logistics cost can be approximated. The following section is used to describe the characteristics of each player's network, since each player has an individual reverse logistics network. The reverse logistics function per collecting agent is obtained with the help of these characteristics.

The manufacturer uses a part of the manufacturing plant to perform the recovery processes in this model. Additionally, the plant carries out the coordination and handling associated with the collection of used products. Hence, the fixed cost of operating the facility that is assigned to the coordination and handling of returns is part of the manufacturer's reverse logistics cost function. Only smaller vans with the corresponding volume capacity and mileage cost are installed, since the accessibility of the collection points is limited. Milk-runs are introduced as the main transport mode, due to the difficulty in matching vehicle capacity with the accumulation at the collection points and the necessity of flexible reaction to changing parameters. The sum of these specifications leads to the non-linear reverse logistics cost per unit of the manufacturer ${}^{M}C_N$ that is given by

$$ {}^{M}C_N = \frac{F_M}{A_M\,\rho_M} + 2\,\ell\,\frac{T}{V}\,\rho_M + T\sqrt{\frac{1}{3\rho_M}}\,. \tag{3.22}$$

Plotting this function over all possible values of $\Theta \in [0,1]$ results in Figure 3.6. It can be observed that function (3.22) is continuous and convex resembling economies of scale in quantity as illustrated in Figure 3.2. For generalisation, Figure 3.6 is dimensionless in the distance that covers the market area. Nevertheless, if the distance would vary, the conclusions drawn from Figure 3.1 about economies of scale in distance would be reflected in a similar way. The manufacturer is only able to reach customers up to a certain density. This density φ_M determines the quantity of returns that can be collected through the manufacturer's network and thereby conditions $\rho = \varphi\,\Theta$. It is assumed that the manufacturer does not reach as many customers as the retailer or the third-party. Therefore, the manufacturer can only collect up to 30% of the overall collection volume in this model.

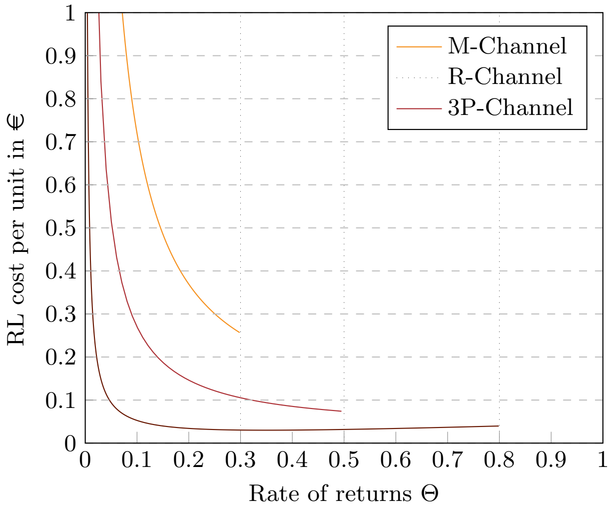


FIGURE 3.6: *The non-linear reverse logistics cost function of M, R and 3P.*

The retailer uses the retail stores as consolidation hubs to ship used products to the manufacturer's recovery facility. Each retail store serves within a service area defining a piece of the overall market area that is supplied with the manufacturer's products. The fixed cost of operating a retail store, that is assigned to the consolidation of returns, is part of the reverse logistics cost of the retailer. By using the retail stores as consolidation hubs, only the tour between customers to the service area is carried out via milk-run transport with smaller vans. Direct shipments with higher capacity vehicles cover the distance between the retail stores and the manufacturer's recovery plant. The back-haul transport from the retail store to the recovery facility can be used to return the used products to the manufacturer, since there is already an existing transport network to supply the retailer with the manufacturer's products. That is why, only the transport cost from the retail store to the manufacturer is included in the second part of the transport cost. Additionally, the retailer is closer to the customers and thus has a higher density φ_R that results in more returns expressed by ρ_R. It is assumed that the retailer's customer density can reach up to 50% of the used products overall in this model. The description of the retailer's logistics network defines the non-linear reverse logistics cost per unit of the retailer ${}^{R}C_N$ that is given by

$$ {}^{R}C_N = \frac{F_R}{A_R\,\rho_R} + \frac{4}{3\sqrt{\pi}}\sqrt{A_R}\,\frac{T}{V}\rho_R + T\sqrt{\frac{1}{3\rho_R}} + \ell\,\frac{t}{v}\rho_R\,. \tag{3.23} $$

Additionally, Figure 3.6 shows the retailer's reverse logistics cost ${}^{R}C_D$, plotted over all values of the return rate. Similar observations as for the manufacturer can be made regarding the shape of function (3.23). However, the reverse logistics cost of the retailer are generally lower than those of the manufacturer resembling the relation between the acquisition cost of the three players.

The third-party logistics service provider has the greatest flexibility in using network structures with consolidation hubs to minimise transport cost. According to Fleischmann et al. [28] there exists an optimal size of service area that can be derived from the first-order conditions of the annualised fixed cost of an operation in equation (3.12) and the cost of milk-run transport in equation (3.20). This optimal service area A^*_{3P} is calculated as

$$ A^*_{3P} = \left(\frac{3\sqrt{\pi}\,F_{3P}\,V}{2\,T\,\rho_{3P}}\right)^{2/3}. \tag{3.24} $$

The equation compares the cost of operating the consolidation hub with the transport cost assigned to the milk-runs and thereby defines an optimal service area. The third-party uses this equation as an indication on the number of consolidation hubs necessary to serve the market area. An optimal service area is configured according to the maximum, since the service facility has to be able to handle the maximum of available returns in the service area. Therefore, the fixed price F_{3P} does not change with area but an optimal service area A^*_{3P} changes. Comparable to the retailer, the third-party uses low capacity vehicles to ship returns via milk-runs from the customer to the hubs and high capacity vehicles to supply the manufacturer via direct shipments with used products. Line-haul and back-haul are included in the direct shipment cost from the consolidation hub to the manufacturer's recovery facility, since a transport network between the third-party and the manufacturer does not exist. Out of the three collecting agents, the third-party is not only the one that has the most flexible network structure, but can also reach the most customers during the collection process due to its high density in φ_{3P}, that determines ρ_{3P}. The customer density of the third-party is assumed to reach the height 80% in this model. The characterisation of the third-parties' logistical structures leads to the non-linear reverse logistics cost per unit of the third-party ${}^{3P}C_N$ that is calculated as

$$ {}^{3P}C_N = \frac{F_{3P}}{A^*_{3P}\,\rho_{3P}} + \frac{4}{3\sqrt{\pi}}\sqrt{A_R}\,\frac{T}{V}\,\rho_{3P} + T\sqrt{\frac{1}{3\rho_{3P}}} + 2\,\ell\,\frac{t}{v}\,\rho_{3P}\,. \tag{3.25} $$

Figure 3.6 shows the non-linear reverse logistics cost ${}^{3P}C_N$ for the scale effects described. Compared to the reverse logistics cost of the manufacturer and the retailer, the cost of the third-party is not only the lowest, but also benefits the most from the economies of scales as the third-party can define an optimal service area.

It is not realistic to assign a fixed cost of operating a facility independently of its capacity, since the total number of returns to be served by a facility is dependent on its capacity. Hence, the cost of opening a facility is a function of its capacity κ_i [90]. As soon as the capacity limit of a facility is reached, a new facility needs to be opened up to handle the overhead returns. Therefore, whenever a facility reaches its limits a jump occurs in the reverse logistics cost function, due to adding the fixed cost of operating an additional facility. That is why, the reverse logistics cost function per player becomes discontinuous.

For the manufacturer, the introduction of a capacity limit κ_M leads to equation 3.26 that describes M's reverse logistics cost function calculated as

$$^{M}C_D = \frac{F_M/(A_M/\kappa_M)}{A_M\,\rho_M}\left\lceil\frac{A_M\,\rho_M}{\kappa_M}\right\rceil + 2\,\ell\,\frac{T}{V}\,\rho_M + T\sqrt{\frac{1}{3\rho_M}}\;. \tag{3.26}$$

The equation plotted over all possible values of the return rate $\Theta \in [0, 1]$ results in Figure 3.7. As the manufacturing plant reaches the capacity limit, the facility has to be extended. This extension leads to a jump at κ_M in the reverse logistics function. Therefore, the reverse logistics function of the manufacturer is not only non-linear but also discontinuous.

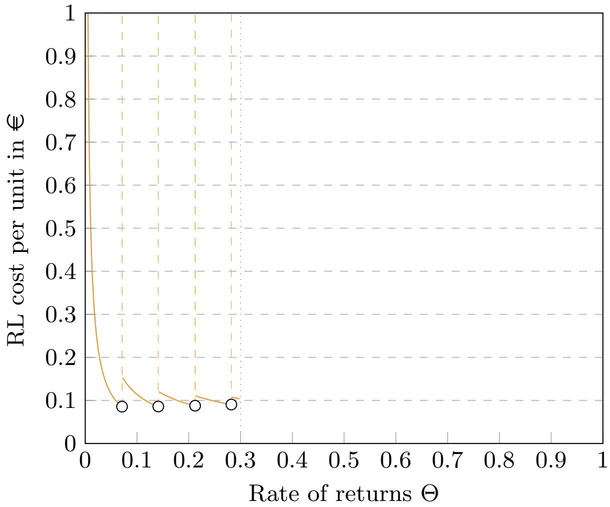


FIGURE 3.7: *The non-linear and discontinuous reverse logistics cost function of the manufacturer.*

The retailer has a lower capacity limit per service facility. As there are more than one service facility inside the manufacturer's market area the overall capacity of all retail stores is comparable to the capacity of the manufacturer's recovery facility. Equation 3.27 shows the manner in which the retailer's reverse logistics cost is computed with the introduction of a capacity limit per service facility κ_R given by

$$^{R}C_D = \frac{F_R/(A_R/\kappa_R)}{A_R\,\rho_R}\left\lceil\frac{A_R\,\rho_R}{\kappa_R}\right\rceil + \frac{4}{3\sqrt{\pi}}\sqrt{A_R}\,\frac{T}{V}\rho_R + T\sqrt{\frac{1}{3\rho_R}} + \ell\,\frac{t}{v}\rho_R\;. \tag{3.27}$$

Plotting the output of this equation over the possible rates of return results in the plot given in Figure 3.8. This plot of the non-linear and discontinuous reverse logistics cost function of the retailer allows to draw a conclusion that is comparable to the manufacturer's function. Nevertheless, the lower capacity limit leads to a higher frequency of jumps in the scope of the collection.

Not only the manufacturer and the retailer are facing a capacity limit of their service facility, but also the third-party has to deal with the limit κ_{3P}. The capacity limit per facility is smaller than the manufacturer's, but the joint capacity of all consolidation hubs is comparable. Equation 3.28 describes the third parties' reverse logistics cost function calculated as

$$^{3P}C_D = \frac{F_{3P}/(A^*_{3P}/\kappa_{3P})}{A^*_{3P}\,\rho_{3P}}\left\lceil\frac{A^*_{3P}\,\rho_{3P}}{\kappa_{3P}}\right\rceil + \frac{4}{3\sqrt{\pi}}\sqrt{A_R}\,\frac{T}{V}\,\rho_{3P} + T\sqrt{\frac{1}{3\rho_{3P}}} + 2\,\ell\,\frac{t}{v}\,\rho_{3P}\;. \tag{3.28}$$

The output of this equation is plotted over $\Theta \in [0, 1]$ to illustrate the non-linear and discontinuous reverse logistics cost, leading to Figure 3.9. The jumps in the function are evenly distributed, due to the flexibility in the optimal size of the third-parties' service area.

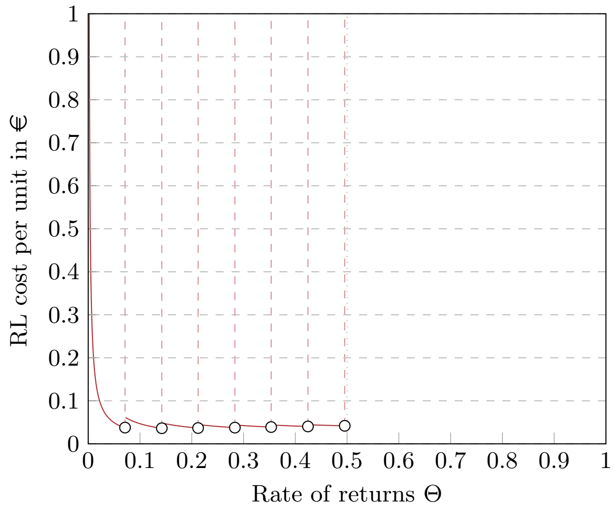


FIGURE 3.8: *The non-linear and discontinuous reverse logistics cost function of the retailer.*

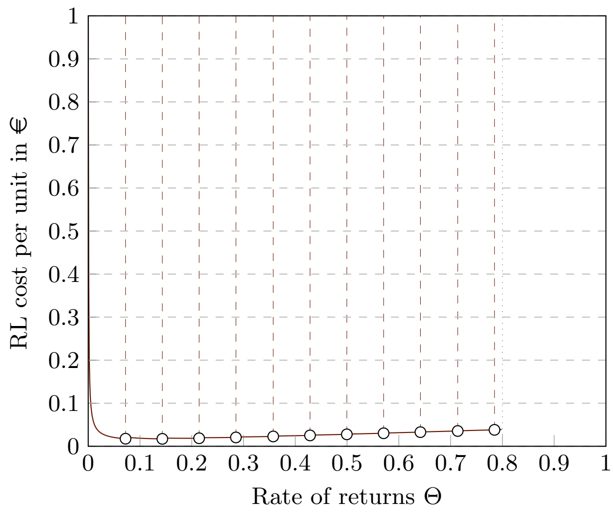


FIGURE 3.9: *The non-linear and discontinuous reverse logistics cost function of the third-party.*

With the introduction of non-linearity and discontinuity of the reverse logistic cost, the formulation becomes more realistic. However, the complexity increases as well. This fact leads to a request for simulation to enable the analysis of the overall model and the interaction between the players in different market scenarios.

3.3 Payoff functions

Every player in this model has an individual payoff function that determines the profit. The profit is derived from revenue minus cost. With linear reverse logistics cost the profit functions are concave in their decision variables. That is why, the optimality of these decision variables is defined by first-order conditions [73]. With the introduction of player-specific reverse logistics cost C_D that is non-linear and discontinuous, an optimal rate of returns Θ_i per player i has to be re-defined with the help of the more advanced payoff functions. The modified reverse logistics cost introduces a customer density φ_i specific to each player i that changes Θ_i to ρ_i according to $\rho = \varphi\,\Theta$. Additionally, the equations (3.26) – (3.28) of the reverse logistics cost are included in the payoff functions of all players. However, the payoff function per player remains concave in the decision variables. Therefore, the optimality can still be retrieved from the first-order conditions.

3.3.1 The manufacturer's payoff

The manufacturer wants to maximise profit by setting the wholesale price w and deciding on the rate of collecting returns. The installations needed to produce products that are released into the market as well as to recover products that have been collected are available at the manufacturer's manufacturing plant. The choice on whether to repair or disassemble a used product depends on the condition of the return. However, the modular design of the product makes recovery easy. Additionally, the materials used to manufacture a product are valuable. In the forward channel the products are distributed via a network of retail stores. The collection of used products in the reverse channel can be carried out by the manufacturer or subcontracted to a retailer or a third-party providing the collection service. Using the parameters described in the market mechanisms of Section 3.1 and a linear ${}^{M}C_A$, let

w	be the wholesale price per unit,
R_M	be the scaling parameter of M per unit, and
${}^{M}C_A$	be the acquisition cost of M per unit.

The objective function of the manufacturer according to Savaskan et al. [73] can be stated as

$$\max_{\Theta_M} {}^{M}\Pi = \frac{\phi - \beta w}{2}[w - c_m + \Theta_M \Delta] - R_M \Theta_M^2 - {}^{M}C_A \Theta_M \frac{\phi - \beta w}{2}\,. \tag{3.29}$$

In case of the manufacturer being the collecting agent, the linear acquisition cost leads to a concave payoff function that is shown in Figure 3.10(a). The payoff function of the manufacturer, illustrated in Figure 3.10(b), results from exchanging the linear acquisition cost constant ${}^{M}C_A$ in equation (3.29) with the non-linear and discontinuous reverse logistics cost function ${}^{M}C_D(\Theta_M)$ from equation (3.26).

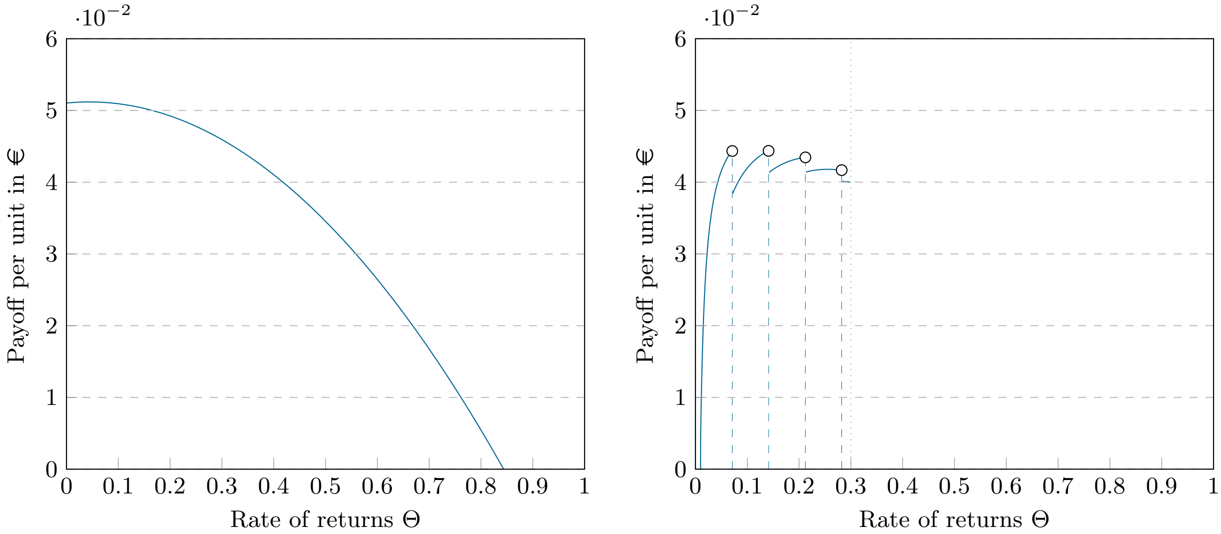
The unique best response wholesale price w can be obtained according to Savaskan et al. [73] by deriving the first-order condition of equation (3.29) given by

$$w = \frac{\phi + \beta c_m}{2\beta} - \frac{(\Delta - {}^{M}C_A)^2(\phi - \beta c_m)}{2[8\,R_M - \beta(\Delta - {}^{M}C_A)^2]}\,. \tag{3.30}$$

Even with changing reverse logistics cost through different return rates, the wholesale price always converges towards the same value. Therefore, the payoff function of the manufacturer changes insignificantly by including a varying w in comparison to a stable wholesale price. Additionally, wholesale prices do not change drastically during a sales season. Therefore, the model works with a set wholesale price w that is deducted by including the linear reverse logistics cost ${}^{M}C_A$ in equation (3.30).

3.3.2 The retailer's payoff

Deciding on the collection rate and setting the retail price p, the retailer tries to maximise profits. The retailer buys products to a given wholesale price from the manufacturer in the forward supply chain. After undertaking the collecting process in the reverse supply chain, the



(a) *M's payoff with linear RL cost* $^{M}C_A$ (b) *M's payoff with discontinuous RL cost* $^{M}C_D$

FIGURE 3.10: *The payoff function of the manufacturer with changing RL cost.*

retailer sells the used products back in exchange for a transfer price b_R. The transfer price is determined by the manufacturer. This payment functions as a compensation for the retailer's collection effort. Let

p	be the retail price per unit,
b_R	be the transfer price of R per unit,
R_R	be the scaling parameter of R per unit, and
$^{R}C_A$	be the acquisition cost of R per unit.

If $^{R}C_A$ is linear, the objective function of the retailer may be calculated according to Savaskan et al. [73] as

$$\max_{\Theta_R} {}^{R}\Pi = (\phi - \beta\, p)(p - w) + b_R\, \Theta_R(\phi - \beta p) - R_R\, \Theta_R^2 - {}^{R}C_A \Theta_R(\phi - \beta p)\ . \tag{3.31}$$

The linear acquisition cost leads to Figure 3.11(a). The payoff function of the retailer including the non-linear and discontinuous reverse logistics cost function $^{R}C_D(\Theta_R)$ from equation (3.27) is plotted in Figure 3.11(b).

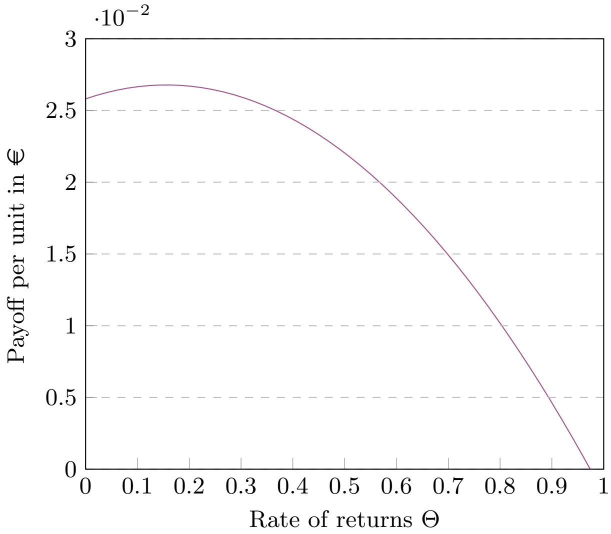
The retailer's unique best response retail price p can be obtained according to Savaskan et al. [73] by deriving the first-order condition of equation (3.31) that is given by

$$p = \frac{\left(\phi + \beta[w - (b_R - {}^{R}C_A)\Theta_R]\right)}{2\beta}\ . \tag{3.32}$$

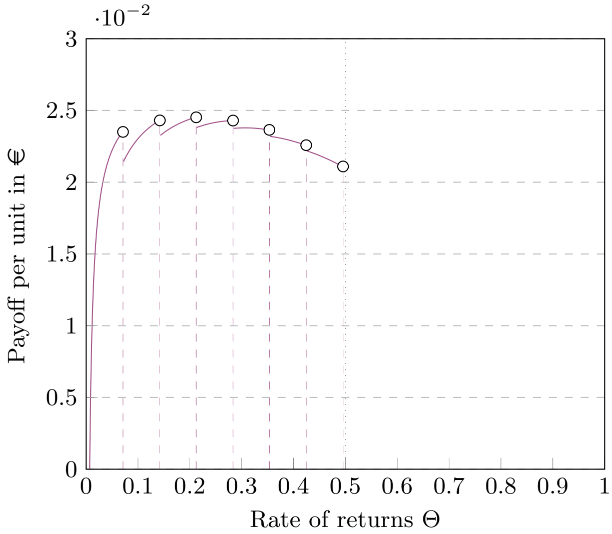
If the retail price is modelled to correspond to changing collection, neither the payoff function with linear nor non-linear and discontinuous reverse logistics function changes significantly. Therefore, p will be modelled as an optimal retail price defined in equation (3.32). Additionally, it encourages the retailer to collect as many returns as possible, according to the observations of Savaskan et al. [73], if the transfer price $b_R = \Delta$, the unit cost saving from recovery.

3.3.3 The third-parties' payoff

The third-party wants to get subcontracted by the manufacturer to carry out the collection of returns and thereby maximise profits influenced to a large extend by the transfer price b_{3P} that



(a) *R's payoff with linear RL cost* ${}^{R}C_A$



(b) *R's payoff with discontinuous RL cost* ${}^{R}C_D$

FIGURE 3.11: *The payoff function of the retailer with changing RL cost.*

is determined by the manufacturer. Besides the possible contractual relationship to perform the collection, there is no other existing incorporation of the third-party into the manufacturer's network. The third-parties' existing logistics network and synergy effects with other clients lower the transport cost of the third-party to a minimum. Let

b_{3P}	be the transfer price of 3P per unit,
R_{3P}	be the scaling parameter of 3P per unit, and
${}^{3P}C_A$	be the acquisition cost of 3P per unit.

If the acquisition price ${}^{3P}C_A$ is linear, the objective function of the third-party, according to Savaskan et al. [73], is given by

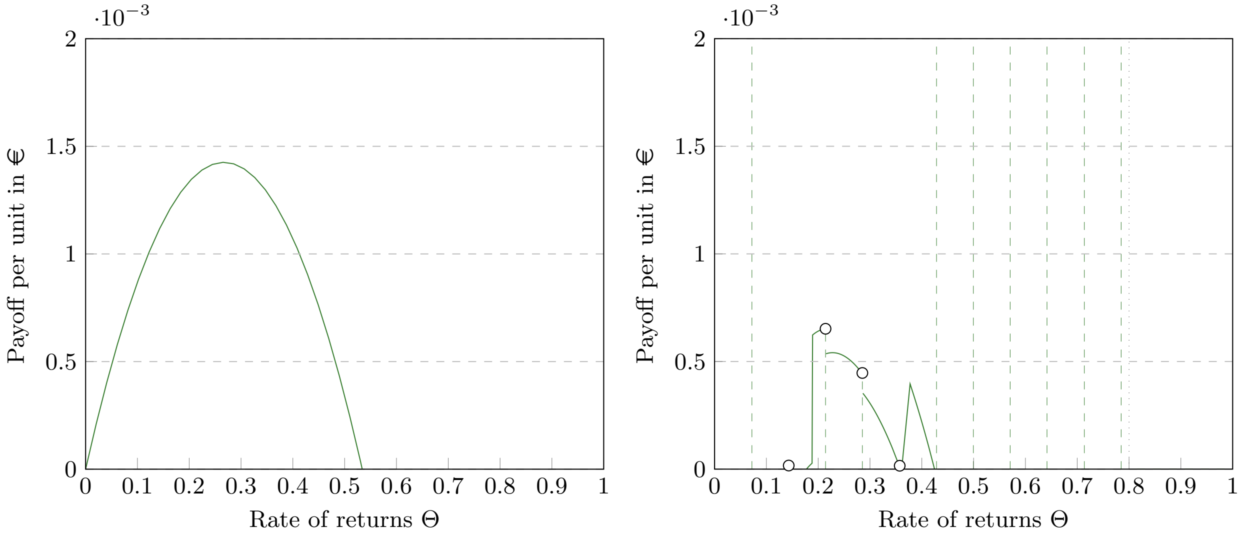
$$\max_{\Theta_{3P}} {}^{3P}\Pi = b_{3P}\,\Theta_{3P}\,(\phi - \beta\,p) - R_{3P}\,\Theta_{3P}^2 - {}^{3P}C_A\,\Theta_{3P}\,(\phi - \beta\,p)\,. \tag{3.33}$$

The objective function of the third party can be plotted over all possible values of the collection rate. With linear acquisition cost ${}^{3P}C_A$ the function leads to Figure 3.12(a), whereas with non-linear and discontinuous reverse logistics cost function ${}^{3P}C_D(\Theta_{3P})$ from equation (3.28) results in Figure 3.12(b).

For the third-party the transfer price is to an large extend dependent on the reverse logistics cost the manufacturer would have to pay to carry out the collection. Therefore, Savaskan et al. [73] formulated an optimal transfer price b_{3P} in their observations given by

$$b_{3P} = \frac{(\Delta + {}^{M}C_A)}{2}. \tag{3.34}$$

Additionally, if the reverse logistics cost of the manufacturer is greater than the cost saving per unit Δ, the manufacturer would pay half the savings to the third-party. This ensures that the transfer price b_{3P} stays positive. The special shape of the reverse logistics cost function ${}^{3P}C_D(\Theta_{3P})$ in Figure 3.12(b) is a result of the influence of the transfer price b_{3P}.



(a) *3P's payoff with linear RL cost* $^{3P}C_A$

(b) *3P's payoff with discontinuous RL cost* $^{3P}C_D$

FIGURE 3.12: *The payoff function of the third-party with changing RL cost.*

3.4 Playing the game

Every player i chooses the amount of collection described by the return rate Θ_i, since the three players compete over the return rate to maximise their overall profits. A benchmark scenario featuring this situation with the non-linear and discontinuous reverse logistics cost for each actor is simulated.

This section makes use of numerical analysis to simulate the situation of conflict. Therefore, algorithms are applied to imitate the player's behaviour. With the help of numerical analysis, the solution of the mathematical model developed in the previous sections is approximated by the production of numbers [68]. The three players i compete over the return rate Θ_i of used products to maximise the individual profit. That is why, the game is modelled as a three-person non-cooperative game in the first place. The cost of collection influences the deduction of payoffs primarily. This cost leads to non-linear and discontinuous functions for different values of the return rate, since it is modelled individually for each player. Each player has an optimal collection rate that results in the player's highest payoff. Every player tries to reach an optimal collection rate to achieve th highest payoff, even when starting with a random collection rate.

An optimal location point of the collection rate Θ is sought-after return rates in an one-dimensional range. The payoff functions per player with continuous cost of collection are concave resulting in a unique optimum per player. The golden ratio search is a technique with zero variance that is applied to find these optima [65]. Therefore, given an unimodal function $\psi(X)$ of a continuous variable X defined on the closed interval $[0, L_n]$, the golden ratio search is used to find the maximum of every player's payoff and thus identifies the stable points of the game. At each step of the search technique the uncertainty is reduced by constant λ. The golden ratio search is a limiting form of the Fibonacci search based on the fact that the constant λ is given by

$$\lambda = \lim_{n\to\infty} \frac{F_n}{F_{n+1}} = \frac{\sqrt{5}-1}{2} \approx 0.618034 \; , \tag{3.35}$$

where F_n and F_{n+1} are successive terms of the Fibonacci sequence. Therefore, the golden ratio search is a simple way to optimise a unimodal function [92].

The golden ratio search proceeds to maximise the unimodal function $\psi(X)$ on the interval $[X_1, X_2]$. Therefore, $\psi(X_3)$ and $\psi(X_4)$ are computed for evaluation in a first step. The search technique obtains function values for point triplets that form a golden ratio, the evaluation points are given by

$$X_3 = X_2 - \lambda(X_2 - X_1) \tag{3.36}$$

and

$$X_4 = X_1 + \lambda(X_2 - X_1)\ . \tag{3.37}$$

The new interval reduced by constant λ is defined by a comparison between the values of $\psi(X_3)$ and $\psi(X_4)$. If $\psi(X_3) > \psi(X_4)$, the interval $(X_4, X_2]$ is discarded. The remaining interval is $[X_1, X_4]$, where X_4 is replaced by X_2 and the new points of evaluation are X_3 and a newly computed X_4 with the use of equation (3.37). The interval $[X_1, X_3)$ is cut out, if $\psi(X_3) < \psi(X_4)$. Within the remaining interval $[X_3, X_2]$ the value X_3 is replaced by X_1. The new points of evaluation are then X_4 and a newly calculated X_3 using equation (3.36). The steps of comparison and update of the intervals are repeated until the desired accuracy is attained. With this procedure, the search technique guarantees that each evaluation of the function $\psi(X)$ brackets the maximum to an interval that is λ-times the size of the preceding interval [67].

Choosing a random value for X and using this value as one of the limits of the interval in which $\psi(X)$ is evaluated, proofs that the search technique ends in the equilibrium point of the game. That is why, initialising the search with one limit randomly chosen and then performing the golden ratio technique results in Algorithm 3.1.

Algorithm 3.1: The golden ratio search.

Input : The payoff for different values of the return rate Θ per player i.
Output: The maximum payoff attainable at a stable return rate Θ per player i.

Initialise c_m, ϕ, β, Δ, w, p, b_R, b_{3P}, ${}^{M}C_N$, ${}^{R}C_N$, ${}^{3P}C_N$, R_M, R_R, R_{3P};
Find a random value for Θ *per player* i;
Calculate payoff for Θ *as* $[0, random, 1]$ *per player* i;
Identify interval limits k, l;
if *Payoff of golden ratio in* $[0, random]$ > *Payoff of golden ratio in* $[1, random]$ **then**
    Set interval $[0, random]$ *to* $[k, l]$;
    else Set interval $[random, 1]$ to $[k, l]$
end
Identify new interval limits m, n;
Initialise $count \leftarrow 0, stop\ count == 3, stopped \leftarrow False$;
while *not stopped* **do**
    if *Payoff for* $m >$ *for* n **then**
        Set l *to* n;
        else Set k to m;
    end
    if *No change in payoffs* **then**
        $count + 1$;
        else $count == 0$;
    end
    if $count == stop\ count$ **then**
        $stopped == True$;
    end
end

The cost of collection analysed in Section 3.2 is directly reflected in the payoff functions described in Section 3.3. The concave payoff function resulting from a continuous cost of collection can easily be searched for its maximum with the golden ratio technique. With the introduction of the discontinuous reverse logistics cost function, the golden ration search is not applicable any longer as it only works with continuous functions. That is why, with the function $\psi(X)$ turning discontinuous, $\psi(X)$ needs to be divided up into positive intervals. Within these intervals the function is continuous. Therefore, the golden ratio search becomes applicable again.

The interactions between the three players is now simulated in the game of a manufacturer producing electronic equipment. The parameters applied to model the game situation are according to Savaskan et al. [73] for the market mechanisms and according to Fleischmann et al. [30] for the reverse logistics cost.

The parameters that describe the game situation are aligned with the assumptions made in Section 3.1. Taken from Savaskan et al. [73], the demand function for the product is linear downward sloping. A real-world product example could be the single-use cameras of Kodak. Retailers that also develop films, send the single-use cameras back to Eastman Kodak Company, where the company uses up to 76% of a used camera for the manufacturing fo a new one. Similar collection activities are undertaken by Canon for print and copy cartridges. However, the general term of electronic equipment is used to describe the product in the simulation to keep the example as universally applicable as possible. Based on Atasu et al. [3] the parameters are chosen to resemble economies and diseconomies of scale. In Table 3.1 all parameters with regards to the market environment are defined.

Description	Parameter	Numerical example
Cost of manufacturing a new product	c_m	€0.5
Cost savings from the re-use per unit	Δ	€0.2
Wholesale price per unit	w	€1.08
Retail price per unit	p	€1.37
Downward sloping linear demand function with		
Positive parameter 1	ϕ	0.5
Positive parameter 2	β	0.3
Scaling parameter per unit for		
Manufacturer	R_M	0.08
Retailer	R_R	0.04
Third-party	R_{3P}	0.02

TABLE 3.1: *The parameter settings for the market mechanisms in the benchmark scenario.*

In 2004, Fleischmann et al. [30] illustrate recycling processes by using the numerical example of an electronic equipment manufacturer adapted from Fleischmann et al. [29] in 2001. For example, OEMs like Canon or Xerox re-manufacture and re-sell used copiers that had been collected from the customers. Using the same equipment for manufacturing and re-manufacturing the recovery process is carried out at the plant of the OEM. The modular design of the copiers allow the re-use of product parts. Additionally, the recovery of printers is another real-world example. This example will again make use of the general example of electronic equipment as a product to allow conclusions that are applicable in various environments. The plant of the OEM is located in Europe. The logistics network is designed around the plant as its centre. It is assumed that the manufacturer serves retailers in 20 major European cities leading to approximately two retailers per 100 km. The inhabitants living within the service area of a retailer create a proportional demand at each retailer. The volume of returned used products is in turn proportional to the volume of sales. The contribution margin of the returns is assumed to be sufficient for the products to be re-manufactured.

Parameters that Fleischmann et al. [30] do not address explicitly are approximated in alignment with the given information. The customer density is approximated in compliance with the amount of customers reached by a collecting agent through the agent's logistics network. The parameters of Fleischmann et al. [30] are scaled down to work in the same dimensions and to be on the same level as the parameters of Savaskan et al. [73]. The cost part of the operation that is responsible for the recycling activities forms the fixed cost per facility, since the focus lies on the reverse supply chain. The capacity per service facility is aligned with the overall market area. All parameters used to compute the reverse logistics cost per player are listed in Table 3.2.

Description	Parameter	Numerical example
Transport cost per km per unit		
Customer - Service facility	T	€ 0.005
Service facility - Manufacturer	t	€ 0.012
Volume per truck		
Customer - Service facility	V	200 m^3
Service facility - Manufacturer	v	800 m^3
Customer density in service area		
Manufacturer	ρ_M	0.3
Retailer	ρ_R	0.5
Third-party	ρ_{3P}	0.8
Fixed cost per service facility		
Manufacturer	F_M	€ 500 000
Retailer	F_R	€ 8 750
Third-party	F_{3P}	€ 1 000
Capacity per service facility		
Manufacturer	κ_M	500 000 m^3
Retailer	κ_M	25 000 m^3
Third-party	κ_M	18 000 m^3
Avg. distance to the manufacturer	ℓ	1 000 km

TABLE 3.2: *The parameter settings for reverse logistics cost in the benchmark scenario.*

The steps to identify the equilibrium points per player are described next. Not only one concave payoff function per player, but each concave payoff function per interval per player has to be included to successfully apply the golden ration search technique.

First of all, the maximum of each interval is found and compared using the Algorithm 3.2. Each interval is searched for positive payoffs. It is assumed that each player would reduce its collection effort before choosing an increase, as it is less costly to use less resources than to invest more. Therefore, if no positive values are found within the interval, the search moves one interval to the left.

Secondly, every maximum of an interval that is found by applying Algorithm 3.2 is used to approximate an envelope function that connects all optimal collection rates and the corresponding payoffs. Every member in a given family of curves is tangent to an envelope curve. A general example describes a family of circles with radius R. Each centre of a circle lies on another circle with radius G. The envelope of this family of circles is then described by a bigger circle of radius $G + R$ and a smaller circle of radius $|G - R|$. A parameter o usually defines the familiy of curves. Intersection occurs between members that differ by a small amount of δo. As δo converges towards zero the locus of the points of intersection form the envelope. Taking the partial derivative with respect to o, equating to zero and eliminating o denotes the function that describes the envelope [60]. The function is described by polynomial approximation, since some points of the envelope function are already known through the maxima. The estimation process of a value y' of the function $f(x)$ for a value x' that for example lies between x_1 and x_2 is called interpolation[61].

Algorithm 3.2: The golden ratio search for discontinuous collection cost.

Input : The payoff for different values of the return rate Θ per player i.
Output: The maximum payoff attainable at a stable return rate Θ per player i.

Initialise c_m, ϕ, β, Δ, w, p, b_R, b_{3P}, ${}^{M}C_D$, ${}^{R}C_D$, ${}^{3P}C_D$, R_M, R_R, R_{3P} φ_M, φ_R, φ_{3P}, F_M, F_R, F_{3P},
κ_M, κ_R, κ_{3P}, A_M, A_R, A_{3P}, T, t, V, $v\,l$, r;
Find a random value for Θ *per player* i;
Identify interval ending and beginning;
Initialise $False$;
while *not stop* **do**
    if *payoff of ending random* < 0 **then**
        Set ending of interval to random $- 1$;
        else Stop $= True$;
    end
end
Initialise $False$;
Set beginning to ending -0.9998;
while *not stop* **do**
    if *payoff of beginning random* < 0 **then**
        Set beginning of interval to random $+ 0.0001$;
        else Stop $= True$;
    end
end
Calculate payoff for tau as beginning and ending of interval per player i;
Identify interval limits k, l;
if *Payoff of golden ratio in* $[0, random] >$ *Payoff of golden ratio in* $[1, random]$ **then**
    Set interval $[0, random]$ *to* $[k, l]$;
    else Set interval $[random, 1]$ to $[k, l]$
end
Identify new interval limits m, n;
Initialise $count, stop\ count, False$;
while *not stopped* **do**
    if *Payoff for* $m >$ *for* n **then**
        Set l *to* n;
        else Set k to m;
    end
    if *No change in payoffs* **then**
        $count + 1$;
        else $count == 0$;
    end
    if $count == stop\ count$ **then**
        $stopped == True$;
    end
end

Approximating a function $f(x)$ over an interval (A, B) with the help of a simpler function like the polynomial $g(x)$ is a branch of numerical analysis. The approximation tries to keep the error between $f(x) - g(x)$ over the interval (A, B) as small as possible. Known values $y_i = f(x_i)$ at points $x_0, x_1, x_2 \ldots x_n$ provide the basis for the approximation. At these points $g(x)$ is chosen to give a zero error. Making the error at intermediate x values as small as possible, can be achieved by piece-wise fitting polynomials P_j to subintervals of (A, B). This results in matching nodes between $P_j(x_j)$ and $f(x_i)$ as well as first derivatives of these functions at the nodes. Splines are piecewise fitted polynomials that pass through nodes and whose first derivatives agree at these nodes [59]. In the computer application, the least-squares metric is used to fit the data values of an optimal return rate of a polynomial of degree 2.

Finally, the interval between the highest payoff obtained by Algorithm 3.2 and the maximum of the envelope functions per player is evaluated with the help of the golden ratio technique in its standard form. Therefore, Algorithm 3.1 can be used to find an optimal return rate with the highest corresponding payoff as a result of this three-step procedure.

3.4.1 Non-cooperative game with highest payoff per unit

In a first evaluation, the highest payoff per unit per player with the corresponding return rate per player is determined. The rate of returns Θ cannot exceed a total of 100%. The benchmark model of the game incorporating the parameters described previously, leads to the results of the highest payoff per unit per player represented in Table 3.3. The highest payoff per unit of the manufacturer is €0.04441 achieved through the corresponding return rate of 47% specific to the manufacturer. The manufacturer has a collection density of 30% and thus collects approximately 14% of the total volume of returns available in the market area. The retailer returns approximately 21% of used products, because the retailer's highest payoff per unit of €0.02453 is achieved at a collection rate of 42% with a customer density of 50%. The collection rate specific to the third-party of 27% leads to the third parties' highest payoff per unit of €0.00065. With the highest density of all players of 80%, the third party collects 21% of all returns available to the market area as well. The sum of all return rates results in a total of 56%. Therefore, each player can aim for the collection rate with the highest payoff per unit as the 100% is not exceeded in this non-cooperative version of the benchmark scenario.

	Manufacturer	**Retailer**	**Third-party**
Player specific return rate	47%	42%	27%
Max. payoff per unit	€0.04441	€0.02453	€0.00065
Proportion of returns	14%	21%	21%

TABLE 3.3: *The player's highest payoff per unit in the benchmark scenario.*

With the help of Algorithm 3.1 searching the optima per player between the highest payoff per unit obtained with the maximum of the envelope function and Algorithm 3.2, the game can be simulated as depicted in Figure 3.13. This simulation shows the convergence towards the same results that have been discussed and displayed in Table 3.3. In this specific case, the retailer and the third-party have the same proportion on the rate of returns.

3.4.2 Non-cooperative game with highest payoff over the entire market area

So far, all of results focused on the payoffs per unit. What if this concentration shifts towards the profit achievable in the entire market area with a certain price per unit at a specific rate? For the general mathematical model that answers this question let

- P_i be the payoff per unit of player i,
- u_i be the units attainable to player i in the specific interval, and
- Θ_i be the variable that describes the rate of return of player i in the specific interval.

Then the maximum payoff per market area can be calculated by

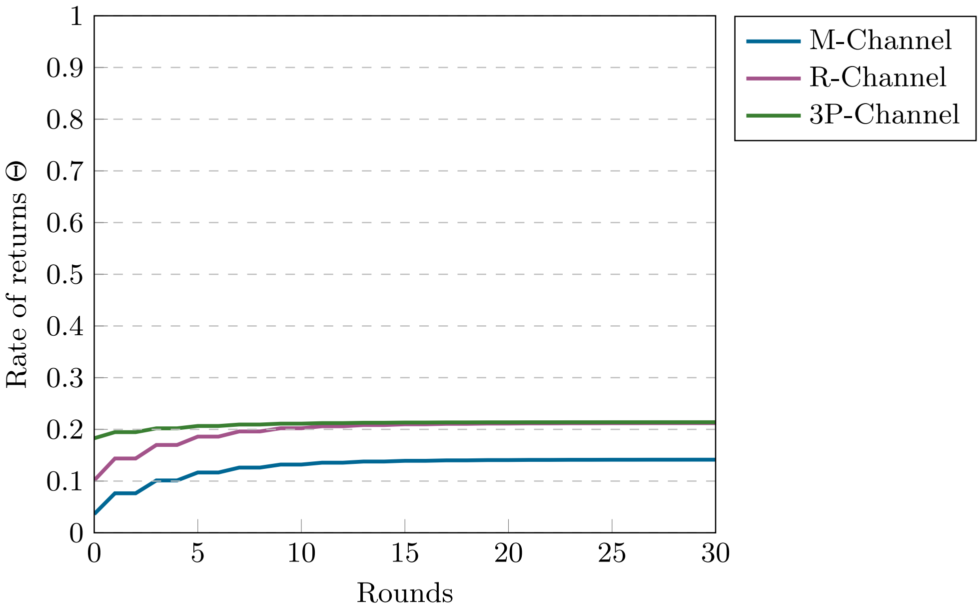
$$\max \sum P_i u_i \Theta_i \ . \tag{3.38}$$

According to the payoff function per unit of the manufacturer in Figure 3.10(b) there are four different intervals the manufacturer could operate in. The sum of all payoffs per unit is highest in interval four, where the manufacturer specific rate of collection is at 85% translating to

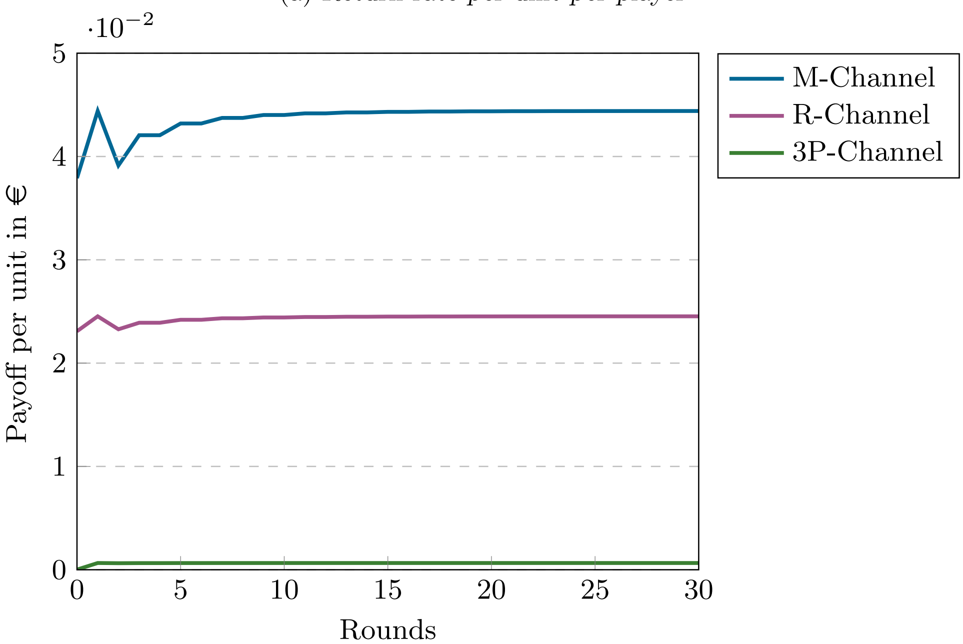
	Manufacturer	**Retailer**	**Third-party**
Player specific return rate	85%	85%	27%
Max. payoff per area	€75 704.15	€66 617.57	€986.21
Proportion of returns	26%	42%	21%

TABLE 3.4: *The player's highest payoff per area in benchmark scenario.*

26% of the entire market area. Therefore, the highest payoff results in €75 704.15 per area for the manufacturer. For the retailer, operating in seven different intervals, as displayed in Figure 3.11(b), the highest payoff per area is computed in interval seven. In this interval, the specific collection rate is at 85% for the retailer, comparable to the manufacturer. The 42% of collection in the market area leads to a payoff of €66 617.57 per area for the retailer. Both players move away from their highest payoff per unit to receive an even higher payoff per area with more returns collected to a lower level of payoff per unit. However, the third-party has five profitable intervals corresponding to Figure 3.12(b). Therefore, the collection within the third interval keeps achieving the highest payoff for the third-party. That is why, the third-parties' specific rate of collection stays at 27% or 21% overall collection rate for the entire market area. The third-party receives a payoff per area of €986.21 in the third interval. Even though, the third-party would achieve a slightly higher payoff per area if the collection would be carried out in the sixth interval, it is unlikely that the third-party would open up two new facilities, making a loss, before opening up service facility number six and thus increasing the payoff. Additionally, the third-party collecting 38% in interval six would exceed the collection rate limit of 100% for all players. However, the manufacturer would collect up to the preferred return rate first as being the Stackelberg leader. It is assumed that the retailer has a greater market power than the third-party, because the retailer invests more in advertising for the collection reflected by a higher scaling parameter. Therefore, the retailer would collect up to the preferred collection level as well. This leaves the third-party with the collection in interval three. As illustrated in Table 3.4, all collection rates summed up result in 89% and thus do not exceed the 100%-limit.



(a) *Return rate per unit per player*



(b) *Payoff per unit per player*

FIGURE 3.13: *The simulation of the benchmark scenario.*

CHAPTER 4

Sensitivity and scenario analysis

Contents

The non-cooperative game is analysed with regards to stability of the parameters as well as the outcomes in different scenarios. Therefore, the parameters originating from Savaskan et al. [73] and Fleischmann et al. [30] are changed slightly. A sensitivity analysis is carried out on the parameters that describe the categories customer density, transport and fixed cost of operating a service facility. Different scenarios with focus on the logistics network of the players as well as the influence of a small or a large market area are evaluated in the scenario analysis.

4.1 Sensitivity of the parameters

The effect of changes per parameter in the market environment determines how sensitive the payoffs in the game are to the input. Therefore, a sensitivity analysis assesses the robustness of results in regards to applied data and uncertain assumptions [48]. The question of what happens to an optimal solution if the coefficients of the non-linear and discontinuous problem vary is investigated with the help of this analysis. In answering this question not only the results of changes in conditions, but also the possibilities of influencing the game become apparent [79].

The sensitivity analysis becomes a crucial part in the methodology because the status of an optimal solution in the game cannot be understood correctly without it. Fiacco [24] demonstrates with the following example that even the solution of very simple mathematical problems differ slightly or drastically for minor perturbations within the parameters. The problem

$$\min \varepsilon x \tag{4.1}$$

$$\text{subject to}\, x \geq -1 \tag{4.2}$$

is solved by $x(\varepsilon) = -1$ if $\varepsilon > 0$, $x(\varepsilon)$ as any chosen value in $[-1, \infty)$ if $\varepsilon = 0$ or with no finite solution for $\varepsilon < 0$. An optimal objective function value of this problem has no lower bound for the values of ε and is given by $f^*(\varepsilon) = -\varepsilon, 0$. Therefore, even with small variations of ε the solution of the problem can be finite and unique, unbounded or solved by infinitely many solutions. That is why, an overview of techniques in non-linear programming are presented that provides conditions under which solutions well-behave locally and solution properties are estimated as functions of parameters. Differences in the optimal objective function value with variations in a parameter that appears in the right-hand side of the constraints are usually analysed through the theory of point-to-set-maps. "A point-to-set-map Ω from a set $\mathcal{X}$ into a set $\mathcal{Y}$ is a map which associates a subset of $\mathcal{Y}$ with each point of $\mathcal{X}$. Equivalently, Ω can be viewed as a function from the set $\mathcal{X}$ into the power set $2^{\mathcal{Y}}$" [47, p. 591]. Analysing the rate of change for an optimal value function at a solution point leads to the stability properties of this function. There are several forms of implicit function theorems that deal with the problem of solving equations of the form $\phi(x, y) = 0$ in terms of x and y that are applied in functional analysis. There are also approaches that obtain bounds of parametric solutions and errors to not only address the continuity or differential properties of an optimal value [24].

The three groups of parameters that can influence the outcome of the game primarily are density, variable and fixed cost. The sensitivity of the objective functions is analysed per parameter in which each of these components is slightly altered. The new results are then compared to the outcome of the benchmark scenario to draw conclusions about the influence of each component for the purpose of the analysis.

4.1.1 Changes in customer density

The changes in customer density parameters are analysed first. In manufacturing, a high density can involve access to a greater pool of resources or customers. In consumption, an increase in density can lead to a higher accessibility in goods and services. Not only shorter distances, but also the increase in access points favours a high density. Therefore, benefits can be derived from the decreasing cost of accessing different features with increasing density [69]. In the original model of the game, the density differs for each player. The underlying assumption is that within a specific market area the density of customers that can be reached through the different collecting agents is the highest for the third-party followed by the retailer and the lowest for the manufacturer. A reason lies in the difference of the logistical networks of the players. For example, customers of Dell wait several days to get their product, since Dell manufactures and distributes directly. On the other side, Hewlett Packard distributes products indirectly through retailers. Therefore, customers of Hewlett Packard are able to get their product immediately from the store [11]. The density levels of the different players were set in accordance with these examples and logical reasoning, since there is no exact data available. Therefore, the third-parties' customer density is set to 80%, resembling the dense network that is used to serve not only one but various clients and thus reaching the most customers. A network with 50% customer density that is influenced by the structure of the retail stores is made available to the retailer. The manufacturer only has the network available that had been created solely for collection purposes. Therefore, the density of the manufacturer's network is set to the lowest level of 30%. Not only a sensitivity, but also a scenario analysis is performed next to experiment with the level of the customer density parameters.

An increase in 1%, followed by 5% and 10% is inflicted on each of the collecting agents. Compared to the benchmark scenario, the payoff per unit and area per player does not change unless the volume of returns exceeds the capacity of the service facilities. The individual collection rates decrease with an increase in customer density. These changes are displayed in Table 4.1. The manufacturer and the retailer steadily decrease their individual collection rate. With an increase in customer density by 1% and 10% the manufacturer decreases the player specific return rate by approximately 1% and 12% respectively. The retailer decreases the player specific return rate by approximately 7% with the retailer's customer density is set to 60%. However, the third-party obtains a lower payoff per unit as well as a lower payoff per area with an increase in customer density while the rate of collection rises. The number of service facilities the third-party has to open up is dependent on the customer density. Therefore, a higher customer density might induce the opening of a new facility allocating more fixed cost to the returns. This explains the variations in payoff of the third-party with a changing density.

	Benchmark	**Density +1%**	**Density +5%**	**Density +10%**
Manufacturer's				
Max. payoff **per unit**	€0.04441	€0.04441	€0.04441	€0.04441
Player specific return rate	47%	46%	40%	35%
Proportion of returns	14%	14%	14%	14%
Max. payoff **per area**	€75 704.15	€75 704.15	€75 704.15	€80 225.66
Player specific return rate	85%	83%	73%	71%
Proportion of returns	26%	26%	26%	28%
Retailer's				
Max. payoff **per unit**	€0.02453	€0.02453	€0.02453	€0.02453
Player specific return rate	42%	42%	39%	35%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€66 617.57	€66 617.57	€66 617.57	€72 729.65
Player specific return rate	85%	83%	77%	83%
Proportion of returns	42%	42%	42%	50%
Third-parties'				
Max. payoff **per unit**	€0.00065	€0.00063	€0.00053	€0.00050
Player specific return rate	27%	26%	26%	35%
Proportion of returns	21%	21%	22%	32%
Max. payoff **per area**	€986.21	€949.71	€822.43	€672.51
Player specific return rate	27%	26%	26%	25%
Proportion of returns	21%	21%	22%	22%

TABLE 4.1: *The effects of an increasing customer density on the payoff and return rate per player.*

The densities are decreased by 1%, 5% and 10% in the second step. The decrease in customer density leads to an increase in player specific return rate as displayed in Table 4.2. Unless the number of returns exceed the service facilities' capacity, the payoff per unit stays the same for all players. The manufacturer and the retailer experience an increase in their collection rates by approximately 24% and 11% for a decreasing density of 10%. The manufacturer has to collect all the returns obtainable for the density of 71% to ensure reaching the highest payoff per area. Additionally, the retailer has to collect 53% for the highest payoff per area.The third-parties' payoff per unit as well as the payoff per area increases with a changing individual return rate. Therefore, the third-party benefits from the low collection rates of the manufacturer and the third-party.

In conclusion, the third-party is effected the most by changes in customer density. Changes result in differences in collection rate and payoff, since the network of the third-party is influenced by the customer density. Therefore, the customer density of the third-party needs to be defined precisely while setting up this model. The manufacturer and the retailer are more flexible when it comes to the choice of the right customer density. Even though the individual collection rate is influenced, the maximum payoff per unit and per area only varies if the service facilities cannot

	Benchmark	Density −1%	Density −5%	Density −10%
Manufacturer's				
Max. payoff **per unit**	€0.04441	€0.04441	€0.04441	€0.04441
Player specific return rate	47%	49%	57%	71%
Proportion of returns	14%	14%	14%	14%
Max. payoff **per area**	€75 704.15	€75 704.15	€65 191.34	€44 403.69
Player specific return rate	85%	88%	79%	71%
Proportion of returns	26%	26%	21%	14%
Retailer's				
Max. payoff **per unit**	€0.02453	€0.02453	€0.02453	€0.02453
Player specific return rate	42%	43%	47%	53%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€66 617.57	€58 005.46	€58 005.46	€51 538.53
Player specific return rate	85%	72%	79%	77%
Proportion of returns	42%	35%	35%	31%
Third-parties'				
Max. payoff **per unit**	€0.00065	€0.00070	€0.00075	€0.00088
Player specific return rate	27%	26%	26%	36%
Proportion of returns	21%	21%	20%	28%
Max. payoff **per area**	€986.21	€1 076.78	€1 046.17	€1 586.55
Player specific return rate	27%	49%	26%	36%
Proportion of returns	21%	39%	22%	25%

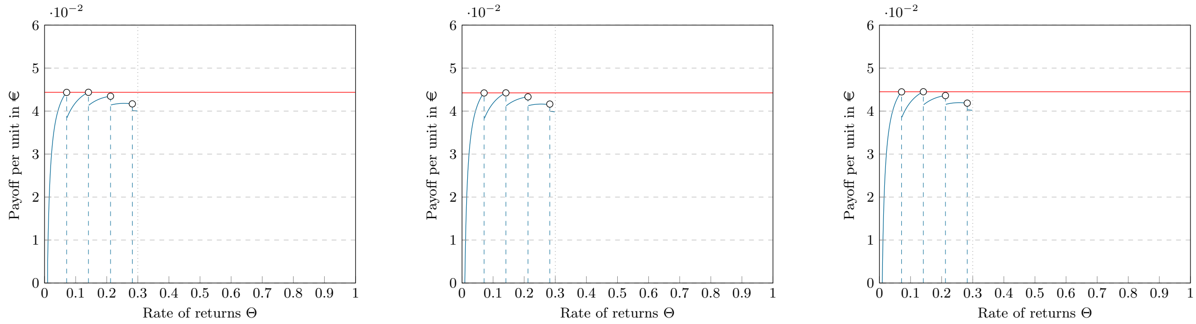
TABLE 4.2: *The effects of a decreasing customer density on the payoff and return rate per player.*

cope with the volume of returns. In general, the higher the customer density, the more returns can be collected resulting in an increase in payoff per player.

4.1.2 Changes in transport cost

The variable component of the cost of collection that is charged for transport activities is influenced by the fuel price. What happens if the fuel price increases or decreases by 1%, 5% and 10% due to changes in the economy? All effects will be compared to the benchmark scenario in Section 3.4 with milk-run transport cost of €0.005 per km per unit and a direct shipment transport cost of €0.012 per km per unit to draw conclusions and adapt the configuration of the model. The cost of collection is dependent on the distance travelled and thus the fuel price as well as the truck capacity. The low-volume truck that carries out the milk-run transport charges €0.000025 per unit when fully loaded. The high-volume truck is used for direct shipments with full-truck load and assigns €0.000015 to every unit. The cost of milk-run and direct shipment transport determine the variable part of the cost of collection.

An increase of 1% in fuel price would result in a cost for milk-run transport of €0.00505 and of €0.01212 for direct shipment. The price of fuel rising by 5% and 10% results in €0.00525 and €0.0126 as well as €0.0055 and €0.0132 for milk-run and direct shipment respectively. The effects of the different steps of increasing fuel price are displayed in Table 4.3. With a rising fuel price, the payoff per player per unit as well as per area decreases. The rate of collection to obtain the highest payoff per player stays at a constant level. However, the third-party has to adjust the rate of return more vividly, since the logistics network of the third-party strongly depends on the balance between cost of opening up a new service facility and transport cost. Illustrated in Figure 4.1(b), the manufacturer's highest payoff per unit decreases by €0.00013, while the payoff per area is €562.03 with an increase in fuel price of 10%. The retailer's highest payoff per unit, depicted in Figure 4.2(b), as well as the payoff per area decreases by €0.0001 and €415 respectively. The 10% increase leads to a decrease of €0.00014 for the third-parties'



(a) *M's payoff in benchmark* (b) *M's payoff with fuel +10%* (c) *M's payoff with fuel −10%*

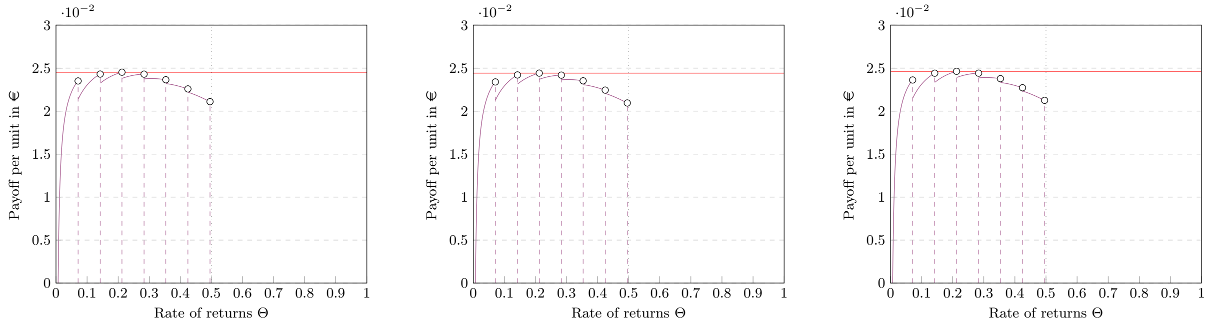
FIGURE 4.1: *The effect of a changing fuel price on M's payoff per unit.*

highest payoff per unit, illustrated in Figure 4.3(b), and € 190.33 for the payoff per area.

	Benchmark	**Fuel** +1%	**Fuel** +5%	**Fuel** +10%
Manufacturer's				
Max. payoff **per unit**	€ 0.04441	€ 0.04439	€ 0.04434	€ 0.04428
Player specific return rate	47%	47%	47%	47%
Proportion of returns	14%	14%	14%	14%
Max. payoff **per area**	€ 75 704.15	€ 75 647.71	€ 75 422.50	€ 75 142.12
Player specific return rate	85%	85%	85%	85%
Proportion of returns	26%	26%	26%	26%
Retailer's				
Max. payoff **per unit**	€ 0.02453	€ 0.02452	€ 0.02448	€ 0.02443
Player specific return rate	42%	42%	42%	42%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€ 66 617.57	€ 66 576.07	€ 66 410.07	€ 66 202.57
Player specific return rate	85%	85%	85%	85%
Proportion of returns	42%	42%	42%	42%
Third-parties'				
Max. payoff **per unit**	€ 0.00065	€ 0.00064	€ 0.00057	€ 0.00051
Player specific return rate	27%	27%	28%	27%
Proportion of returns	21%	21%	22%	22%
Max. payoff **per area**	€ 986.21	€ 968.50	€ 892.42	€ 795.88
Player specific return rate	27%	27%	28%	27%
Proportion of returns	21%	21%	22%	22%

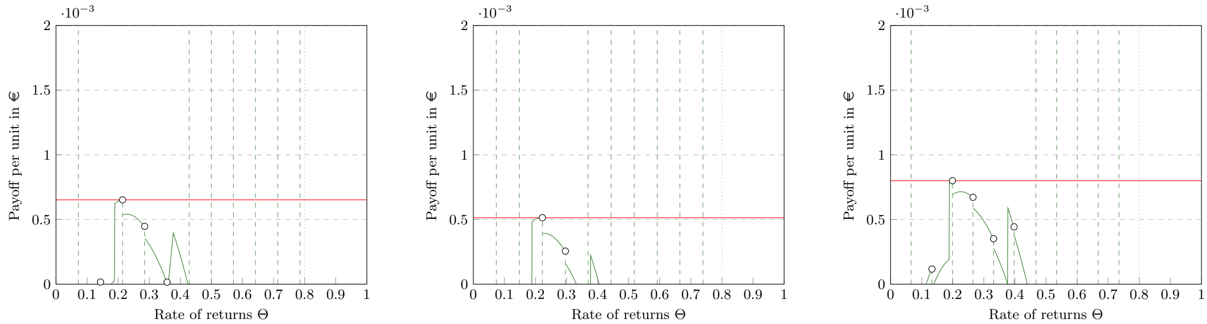
TABLE 4.3: *The effects of an increasing fuel price on the payoff and return rate per player.*

A decreasing fuel price by 1%, 5% and 10% results in a milk-run transport cost of € 0.00495, € 0.00475 and € 0.0045 respectively. Additionally, the direct shipment cost decreases by 1% to € 0.01188, by 5% to € 0.0114 and by 10% to € 0.0108. A decrease in fuel price lowers the cost of transport directly. That is why, the players' payoffs per unit as well as per area increase as shown in Table 4.4. The collection rate per player of the manufacturer and the retailer is not influenced by the fuel price. Depicted in Figure 4.1(c), the manufacturer increases the highest payoff by € 0.00013 per unit if the fuel price decreases by 10%. Additionally, the manufacturer's payoff per area is € 567.15 higher than before. The retailer's payoff per unit increases by € 0.00011 as well as the payoff per area rises by € 414.99. The changes in the retailer's payoff per unit are displayed in Figure 4.2(c). The adaptation of the number of service facilities according to the changing transport cost is responsible for the third-parties' variation in return rate resulting in an increase of payoff per area of € 154.43 in the 10%-example. The payoff per unit of the third-party increases steadily with a decreasing fuel price. Illustrated in Figure 4.3(c), the third-parties' payoff per unit increases by € 0.00015 with a 10% fuel price decrease. Therefore, the



(a) *R's payoff in benchmark* (b) *R's payoff with fuel +10%* (c) *R's payoff with fuel −10%*

FIGURE 4.2: *The effect of a changing fuel price on R's payoff per unit.*



(a) *3P's payoff in benchmark* (b) *3P's payoff with fuel +10%* (c) *3P's payoff with fuel −10%*

FIGURE 4.3: *The effect of a changing fuel price on 3P's payoff per unit.*

decrease in fuel price mirrors the observations on the increase in fuel price for all players. The lower the fuel price, the higher the payoffs per player.

	Benchmark	**Fuel** −1%	**Fuel** −5%	**Fuel** −10%
Manufacturer's				
Max. payoff **per unit**	€0.04441	€0.04442	€0.04447	€0.04454
Player specific return rate	47%	47%	47%	47%
Proportion of returns	14%	14%	14%	14%
Max. payoff **per area**	€75 704.15	€75 760.63	€75 987.08	€76 271.30
Player specific return rate	85%	85%	85%	85%
Proportion of returns	26%	26%	26%	26%
Retailer's				
Max. payoff **per unit**	€0.02453	€0.02454	€0.02458	€0.02464
Player specific return rate	42%	42%	42%	42%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€66 617.57	€66 659.07	€66 825.06	€67 032.56
Player specific return rate	85%	85%	85%	85%
Proportion of returns	42%	42%	42%	42%
Third-parties'				
Max. payoff **per unit**	€0.00065	€0.00068	€0.00073	€0.00080
Player specific return rate	27%	26%	26%	25%
Proportion of returns	21%	21%	21%	20%
Max. payoff **per area**	€986.21	€993.91	€1 061.69	€1 140.64
Player specific return rate	27%	26%	26%	28%
Proportion of returns	21%	21%	21%	23%

TABLE 4.4: *The effects of a decreasing fuel price on the payoff and return rate per player.*

A changing fuel price influences the cost of transport for both transport options, milk-run transport and direct shipping, and for every player directly. It is significant to initialise a valid transport cost to the model, since it changes the level of payoff per player. With an increase in fuel price the highest payoffs attainable are lower compared to the benchmark scenario, while with a decrease in fuel price the payoffs rise. For the manufacturer and the retailer, the rates of collection to achieve the highest payoff stay constant. However, the third-party decides on the number of service facilities by balancing out operational cost and transport cost. Therefore, it is crucial for the third-party to work with accurate fuel prices to be able to design an optimal logistics network.

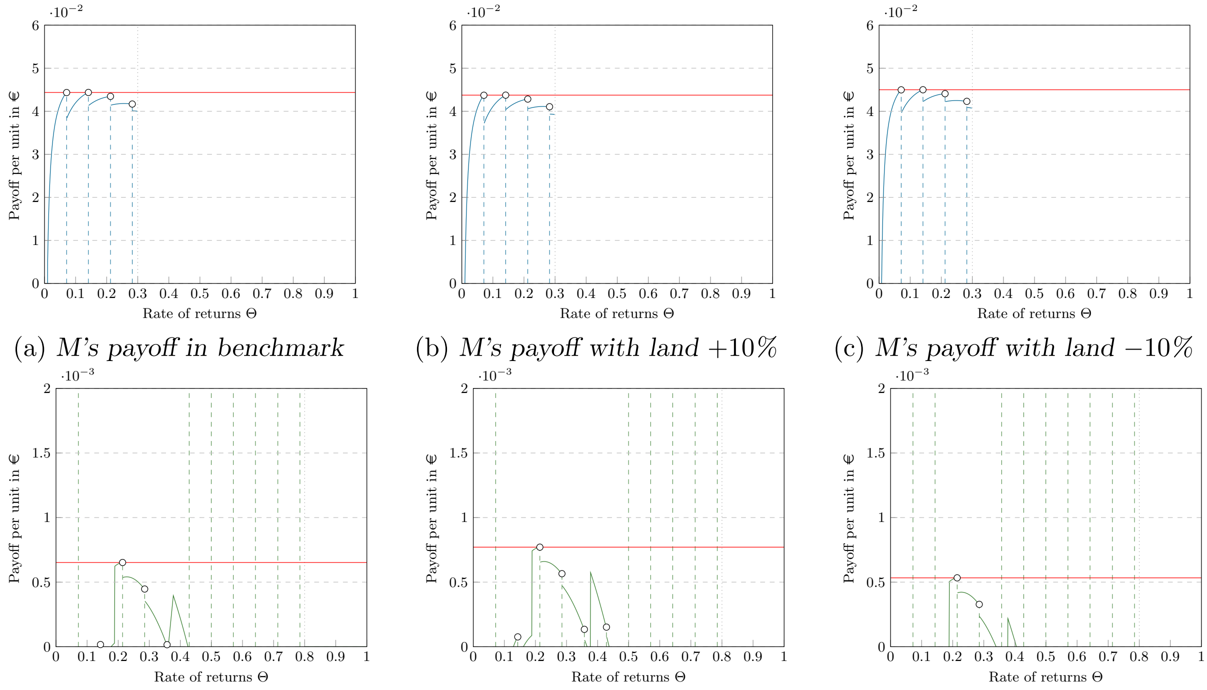
4.1.3 Changes in fixed cost

The effects caused by changes in fixed cost is analysed as a third aspect. The annualised fixed cost for operating a facility are aligned with Fleischmann et al. [30], namely that the cost for operating the part of the facility that is carrying out the collection is included in the reverse logistics cost.

The manufacturer as well as the retailer have an existing network structure that is suitable for their forward supply chains. That is why, the location of the service facilities, the plant for the manufacturer and the retail stores for the retailer, are at fixed locations. What would happen if the price of land increases in the area where these service facilities are located? A reason for the increase of land price can be found in opportunity costs due to limited space. Additionally, the land price can rise due to favourable developments like for example an upgrade of the infrastructure in the surrounding area. The effects of increasing land price for a specific area is negligible for the third-party, since the third-parties' network is flexible in the locations of service facilities. The two scenarios in which the land price of the manufacturer and the land price of the retailer change are investigated. The manufacturer's land price will increase by 1%, 5% and 10% from €500 000 in the benchmark scenario. Afterwards, the manufacturer's land price will be decreased by 1% to 10% to further analyse the sensitivity of a changing fixed cost. A similar example will increase and decrease the annualised fixed cost of operating the service facility of the retailer from €8 750 in the benchmark scenario.

The increase and decrease in price of the manufacturer's land does not only influence the manufacturer, but also the third-party. The third-parties' payoff function is also effected by changes, since the transfer price b_{3P}, that the third-party receives for collecting a unit, depends on the reverse logistics cost the manufacturer would pay if the unit was collected in the manufacturer's channel. The coherence of the two functions is described in equation (3.34).

In the first case, the manufacturer's land price increases and thus the fixed cost of operating the recovery facility is €505 000 for a 1%, €525 000 for a 5% and €550 000 for a 10% increase. The effects of the increase in land price are listed in Table 4.5. With an increase in land price of 10%, the manufacturer's payoff per unit decreases by €0.00063. The decrease in payoff per unit is illustrated in Figure 4.4(b). However, the 10% increase in the manufacturer's land price results in an increase of €787.11 in the payoff per area. The manufacturer increasing the collection rate due to the decreasing payoff per unit is responsible for this increase. The retailer's payoff or collection rate is not influenced by the increase in land price of the manufacturer in the non-cooperative game. Even though the manufacturer's payoff per unit decreases, the third-parties's payoff per unit increases as displayed in Figure 4.4(e). The increase in fixed cost benefits the third-party, since the manufacturer is willing to subcontract the collection effort for a higher b_{3P} to the third-party as the cost to collect a unit becomes more expensive for the manufacturer. The higher transfer price outweighs the cost of collection and thus results in an increase in



(a) *M's payoff in benchmark* (b) *M's payoff with land +10%* (c) *M's payoff with land −10%*

(d) *3P's payoff in benchmark* (e) *3P's payoff with land +10%* (f) *3P's payoff with land −10%*

FIGURE 4.4: *The effects of a changing land price of M on the payoff per unit of M and 3P.*

the third-parties' payoff per unit. Therefore, a 10% increase in the manufacturer's land price leads to an increase in the third-parties' payoff per unit and payoff per area by €0.00012 and €179.55 respectively. In conclusion, an increase in the manufacturer's fixed cost does not only influence the highest payoff per unit attainable for the manufacturer negatively, it also benefits the collection channel of the third-party. Nevertheless, an increase in return rate does lead to higher payoffs per area for the manufacturer.

	Benchmark	**M's land+1%**	**M's land +5%**	**M's land +10%**
Manufacturer's				
Max. payoff **per unit**	€0.04441	€0.04434	€0.04409	€0.04378
Player specific return rate	47%	47%	47%	47%
Proportion of returns	14%	14%	14%	14%
Max. payoff **per area**	€75 704.15	€75 790.07	€76 117.33	€76 491.26
Player specific return rate	85%	86%	87%	88%
Proportion of returns	26%	26%	26%	26%
Third-parties'				
Max. payoff **per unit**	€0.00065	€0.00066	€0.00071	€0.00077
Player specific return rate	27%	27%	27%	27%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€986.21	€1 004.17	€1 075.98	€1 165.76
Player specific return rate	27%	27%	27%	27%
Proportion of returns	21%	21%	21%	21%

TABLE 4.5: *The effects of an increasing land price of M on the payoff and return rate per player.*

Decreasing the manufacturer's land price reduces the fixed cost of operating the re-manufacturing facility by 1% to €495 000, by 5% to €475 000 and by 10% to €450 000. The manufacturer's payoff per unit increases by €0.00062 as illustrated in Figure 4.4(c) with a land price decreasing by 10%. Additionally, the payoff per area of the manufacturer is decreased by €957.3 with this change as the return rate is lowered. Further effects of changes are displayed in Table 4.6. Depicted in Figure 4.4(f), the decreasing land price does not have any positive effects on the third-parties' payoff function. The decrease in the manufacturer's land price by 10% decreases the third-parties' payoff per unit and area by by €0.00012 and €179.54 respectively. The observations correlate with the insights gained from the increase in land price.

	Benchmark	**M's land−1%**	**M's land −5%**	**M's land −10%**
Manufacturer's				
Max. payoff **per unit**	€0.04441	€0.04447	€0.04472	€0.04503
Player specific return rate	47%	47%	47%	47%
Proportion of returns	14%	14%	14%	14%
Max. payoff **per area**	€75 704.15	€75 616.52	€75 248.52	€74 746.85
Player specific return rate	85%	85%	84%	83%
Proportion of returns	26%	26%	25%	25%
Third-parties'				
Max. payoff **per unit**	€0.00065	€0.00064	€0.00059	€0.00053
Player specific return rate	27%	27%	27%	27%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€986.21	€968.26	€896.44	€806.67
Player specific return rate	27%	27%	27%	27%
Proportion of returns	21%	21%	21%	21%

TABLE 4.6: *The effects of a decreasing land price of M on the payoff and return rate per player.*

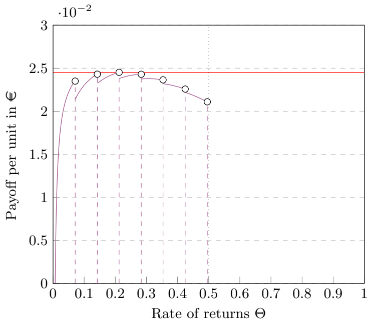
In the second case, the retailer's cost of operating the service facility is increased by a rise in land price. The fixed cost of the retailer of €8 750 is increased by 1% to €8 838, by 5% to €9 188 and by 10% to €9 625. This increase only effects the retailer in the non-cooperative game. The changes are illustrated in Table 4.7. A 10% increase in land price decreases the retailer's payoff to €0.00022 per unit, depicted in Figure 4.5(b), and €771.2 per area, since the return rate stays constant.

	Benchmark	**R's land+1%**	**R's land +5%**	**R's land +10%**
Retailer's				
Max. payoff **per unit**	€0.02453	€0.02451	€0.02442	€0.02431
Player specific return rate	42%	42%	42%	42%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€66 617.57	€66 540.01	€66 231.53	€65 846.37
Player specific return rate	85%	85%	85%	85%
Proportion of returns	42%	42%	42%	42%

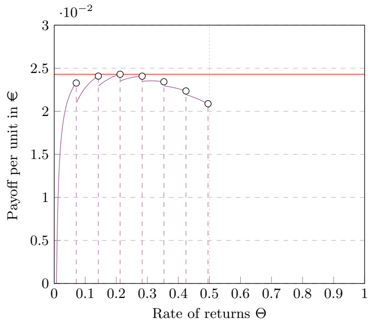
TABLE 4.7: *The effects of an increasing land price of R on the payoff and return rate per player.*

A decreasing land price results in a decreasing cost of operating the service facility of the retailer. Therefore, the retailer's fixed cost is decreased by 1%, 5% and 10% resulting in €8 663, €8 313 and €7 875 respectively. The changes in the highest attainable payoff of the retailer are described in Table 4.8. Resembling the conclusions drawn from the increase in land price, a 10% decrease causes the the retailer's payoff per unit to rise by€0.00022. This increase is illustrated in Figure 4.5(c). The retailer's payoff per area increases by €771.19 with the retailer's land price by decreasing 10% and a constant return rate.

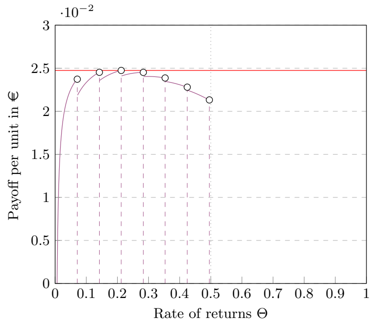
Illustrated in Figure 4.4 and Figure 4.5, the shape of the payoff functions per unit of the manufacturer and the retailer change in the level of payoff attainable. An increase in land price of the manufacturer might increase the transfer price for the third-party, since the collection through the third-party becomes more economic. Nevertheless, an increase in land price of the manufacturer or the retailer causes a decrease in the highest payoff per unit. Comparing the



(a) *R's payoff in benchmark*



(b) *R's payoff with land +10%*



(c) *R's payoff with land −10%*

FIGURE 4.5: *The effects of a changing land price of R on the payoff per unit of R.*

	Benchmark	**R's land−1%**	**R's land −5%**	**R's land −10%**
Retailer's				
Max. payoff **per unit**	€0.02453	€0.02455	€0.02464	€0.02475
Player specific return rate	42%	42%	42%	42%
Proportion of returns	21%	21%	21%	21%
Max. payoff **per area**	€66 617.57	€66 694.25	€67 002.72	€67 388.76
Player specific return rate	85%	85%	85%	85%
Proportion of returns	42%	42%	42%	42%

TABLE 4.8: *The effects of a decreasing land price of R on the payoff and return rate per player.*

observation of the retailer's payoff reduction to the manufacturer's, the decrease in the highest payoff per unit is less drastic for the retailer. However, in terms of the highest payoff per area the levels of the manufacturer increase with a rise and changing return rate while the retailer decreases with an increase in land price and a constant return rate. Additionally, a decrease in the manufacturer's land price results in a decrease in payoff per area with a decreasing collecting rate, while the retailer's decrease in land price results in an increase of the highest payoff per area at a constant collection rate. In the non-cooperative version of the game, the changes in land price only effect the player whose fixed cost varies. The fixed cost influences the attainable level of payoff and should thus be chosen carefully to set up a valid model.

4.2 The parameters in different scenarios

Scenario compared to sensitivity analysis does not only investigate what happens to the outcome of the model if one parameter is changed slightly, but it evaluates what happens if several parameters forming a specific scenario are changed. Significant changes in processes or the environment are evaluated with the help of scenarios. While in scenario planning a variety of possible future scenarios is created to make strategic decisions, this project will make use of scenario analysis to investigate the effects on the model in changing market environments. Especially uncertainties in the business itself or the surrounding environment are used to create scenarios [62].

The benchmark scenario matches a payoff function with every player to determine the player's highest payoff. What happens if the customer density is the same for every player? Are there still differences between the collection channels of the players due to slightly different logistics networks? Additionally, the market area size is changed. How would this change influence the payoff function of each player? Will economies of scale be reached within a smaller market area?

Will diseconomies of scale evolve in a larger market area? Four different scenarios are generated, to answer these questions.

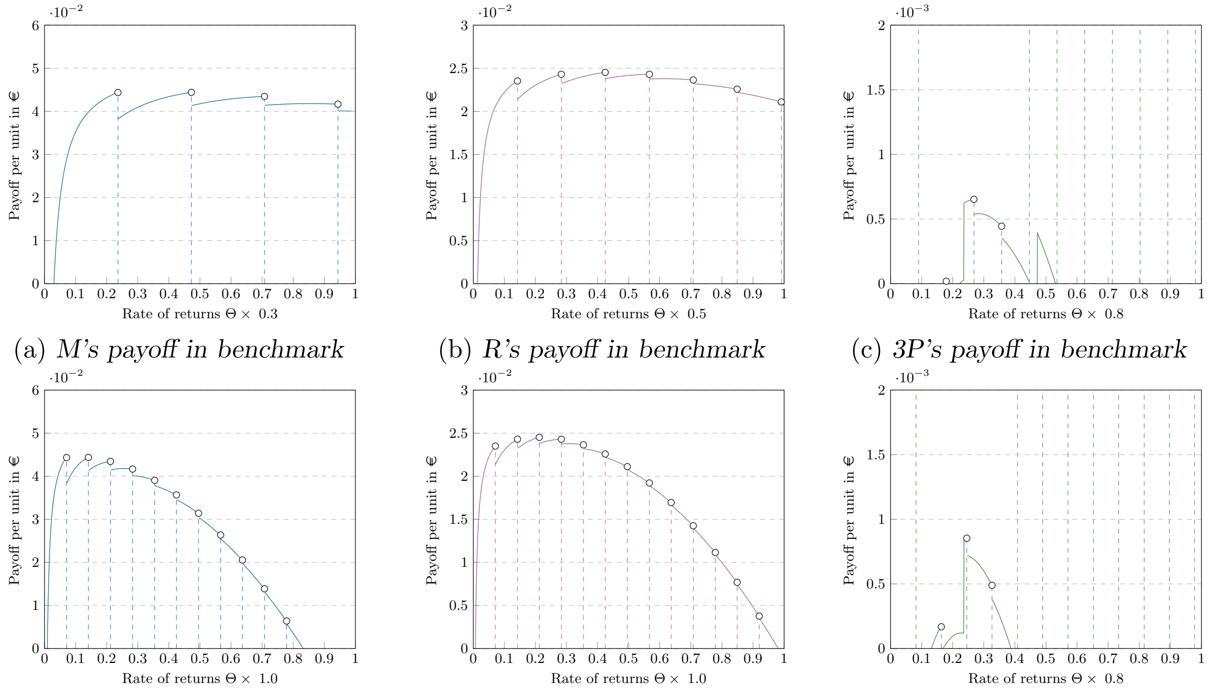
4.2.1 Scenario 1: A high customer density

The question arises on how the benchmark scenario would change, if all players would have the same density parameter. What conclusions can be drawn about the coherence between the collecting effort and the customer density? Every player will be assigned with the same density to answer this question. The extreme case of a 100%-density is evaluated by comparison to the benchmark scenario. Therefore, every collecting agent can reach 100% of the customers within the predefined service area.

A possible scenario could be a collection of returns carried out in Austria, Europe. The population density, describing the number of people in relation to the size of area occupied by these people, is approximately 100% for Austria [84]. All players will be assumed to be able to reach every customer in this area in the generation of this scenario. Therefore, the customer density of each player is set to 100%. Even though Austria's market size is smaller and the land price is slightly higher, all parameters except for the customer density will be set to the levels of the benchmark scenario. Thereby, this scenario focuses on the influence of customer density to the logistics networks of the different players.

Comparing the outcome for the individual density and the density of 100% for each player leads to the following results. The manufacturer's individual density is 30% in the benchmark scenario that is lower than 100%. In Figure 4.6(d) compared to Figure 4.6(a), the manufacturer is still able to obtain a positive payoff for return rates that are greater than 30% up to approximately 82%. For higher collection efforts the payoffs would not be economically viable. The effort of collection needs to be at 47% for the individual density to reach the manufacturer's highest payoff of €0.04441 per unit. This is visualized by Figure 4.6(a). Additionally, Figure 4.6(d) depicts that the individual collection effort of 14% has to be undertaken with a 100%-density to reach the maximum payoff per unit. The density of 100% is higher than the retailer's individual density of 50% in the benchmark scenario. Even with a return rate of approximately 97% the retailer can still attain a positive payoff according to Figure 4.6(e) when compared to the benchmark scenario in Figure 4.6(b). The highest payoff of €0.02453 per unit in the benchmark scenario is achieved by an individual collection effort of 42%. This payoff per unit can also be obtained by 21% in the 100%-scenario as shown in Figure 4.6(e). The third-parties' individual density is given by 80% in the benchmark scenario. Therefore, the collection effort is higher in the case of the 100%-density-scenario. Figure 4.6(f) compared to Figure 4.6(c) depicts that there are greater chances for the third-party to generate a positive payoff with similar densities per player than with individual densities. With a density of 100% the highest payoff of €0.00035 per unit, that is significantly lower than the €0.00065 per unit in the benchmark scenario, is attained by the specific collection effort of 28%. The third-parties' logistics network seems to adapt better to the slightly lower customer density of the benchmark scenario. Table 4.9 summarises that the higher the density, the lower the collection effort needed to achieve the highest payoff per unit for each player.

Table 4.9 does not only show the highest payoff per unit of each player but also displays the highest payoff per area for each collecting agent. The manufacturer shows similar results for the payoff per area as observed for the payoff per unit. As the density increases the highest payoff per area rises by almost a half, while the collection effort decreases by 35% in the 100%-scenario. The conclusion drawn for the manufacturer also applies to the retailer. With a rising density, the effort of collection decreases. Besides, the highest payoff per area of the retailer increases



(a) *M's payoff in benchmark* (b) *R's payoff in benchmark* (c) *3P's payoff in benchmark*

(d) *M's payoff with 100% density* (e) *R's payoff with 100% density* (f) *3P's payoff with 100% density*

FIGURE 4.6: *The effects of a 100% customer density on the payoff per unit per player.*

	Benchmark	**100%-density**
Manufacturer's		
Max. payoff **per unit**	€ 0.04441	€ 0.04441
Player specific return rate	47%	14%
Proportion of returns	14%	14%
Max. payoff **per area**	€ 75 704.15	€ 106 705.44
Player specific return rate	85%	50%
Proportion of returns	26%	50%
Retailer's		
Max. payoff **per unit**	€ 0.02453	€ 0.02453
Player specific return rate	42%	21%
Proportion of returns	21%	21%
Max. payoff **per area**	€ 66 617.57	€ 72 729.65
Player specific return rate	85%	50%
Proportion of returns	42%	50%
Third-parties'		
Max. payoff **per unit**	€ 0.00065	€ 0.00035
Player specific return rate	27%	28%
Proportion of returns	21%	28%
Max. payoff **per area**	€ 986.21	–
Player specific return rate	27%	–
Proportion of returns	21%	–

TABLE 4.9: *The effects of a 100% customer density on the return rate and payoff per player.*

by € 6 112.08. The third-parties' payoff per unit decreases from the 80%- to the 100%-scenario. In the 100%-scenario, the third-party is not able to collect any more since the manufacturer collects 50% as the Stackelberg leader and the retailer follows up to collect the remaining 50%.

Considering Figure 4.7 and comparing the players against each other, every player's specific return rate changes, when the density of all players equally rises to 100%. The manufacturer

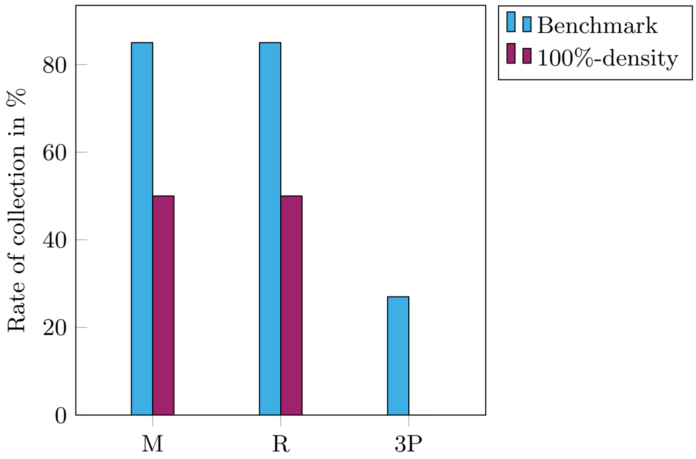


FIGURE 4.7: *The player specific return rate with a 100% customer density.*

and the retailer do not have to put as much effort into the collection as in the benchmark scenario to reach the highest payoff per area that increases. However, the third-party is not able to collect at all in the 100%-scenario. A change in density has great effects on all players as the individual densities are generally lower. With 100% customer density amongst all players, the collection rates of the manufacturer and the retailer converge towards a similar level.

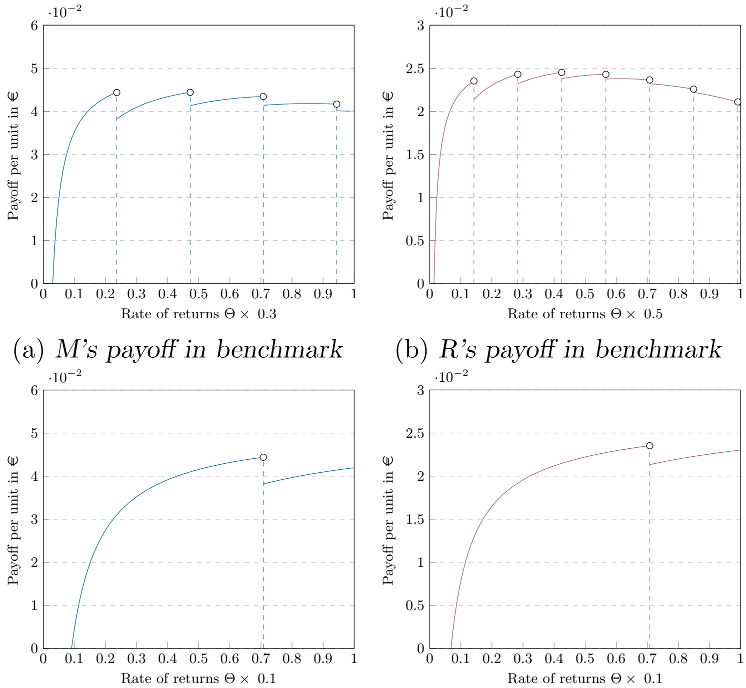
4.2.2 Scenario 2: A low customer density

In the second scenario, the extreme case of a 10% customer density is evaluated and compared to the benchmark scenario. The collection could be carried out in Norway, Europe as a possible example for this scenario. For Norway, the population density is approximately 10% [84]. Therefore, customers are sparely scattered over the market area. Fuel and land price are slightly higher, but to focus on the influence of the customer density, these parameters are set to the level of the benchmark scenario.

The collection for a customer density of 10% is not profitable for the third-party. Neither the payoff per unit nor per area would lead to positive results. Therefore, the third-party is excluded from further comparison.

The manufacturer's payoff per unit with the individual customer density compared to the payoff resulting from the 10%-density is illustrated in Figure 4.8(a) and Figure 4.8(c). Not only the player specific collection rate and thus the shape of the payoff function changes drastically, but also the manufacturer's payoff per unit decreases by €0.00002 as listed in Table 4.10. The individual customer density leads to the payoff per unit for the retailer described in Figure 4.8(b). Compared to the payoff per unit for the 10%-density, the highest payoff drops by €0.001. Additionally, the collection rate of the retailer increases significantly changing the payoff to the function depicted in Figure 4.8(d).

The player specific collection rate increases for the manufacturer's and the retailer's highest payoff per area. However, the highest payoff per area attainable are significantly lower than in the benchmark scenario. Therefore, Figure 4.9 does not describe the differences in player specific collection rate, but the changes in the level of payoff per area. The manufacturer's payoff per area decreases with the customer density of 10% by €53 509.94. The retailer's payoff per area decreases by €54 851.56 comparable to the manufacturer's loss.



(a) *M's payoff in benchmark* (b) *R's payoff in benchmark*

(c) *M's payoff with 10% density* (d) *R's payoff with 10% density*

FIGURE 4.8: *The effects of a 10% customer density on the payoff per unit per player M and R.*

	Benchmark	**10%-density**
Manufacturer's		
Max. payoff **per unit**	€0.04441	€0.04439
Player specific return rate	47%	71%
Proportion of returns	14%	7%
Max. payoff **per area**	€75 704.15	€22 194.21
Player specific return rate	85%	71%
Proportion of returns	26%	7%
Retailer's		
Max. payoff **per unit**	€0.02453	€0.02353
Player specific return rate	42%	71%
Proportion of returns	21%	7%
Max. payoff **per area**	€66 617.57	€11 766.01
Player specific return rate	85%	71%
Proportion of returns	42%	7%

TABLE 4.10: *The effects of a 10% customer density on the return rate and payoff per player.*

The loss of payoff per area are drastic to both, the manufacturer and the retailer. The third-party cannot even carry out a profitable collection in the deserted area of the scenario. The importance of the customer density to the profitability of the collection channel is emphasised.

4.2.3 Scenario 3: A small market area

The average distance ℓ to the manufacturer's recovery facility is reduced from 1 000 km in the benchmark scenario to 200 km to emulate a scenario with a small market area. This change reduces the original market area with a radius of 1 500 km to a market area with a radius of 300 km. A market area with this radius could be located in Germany, Europe for example. The

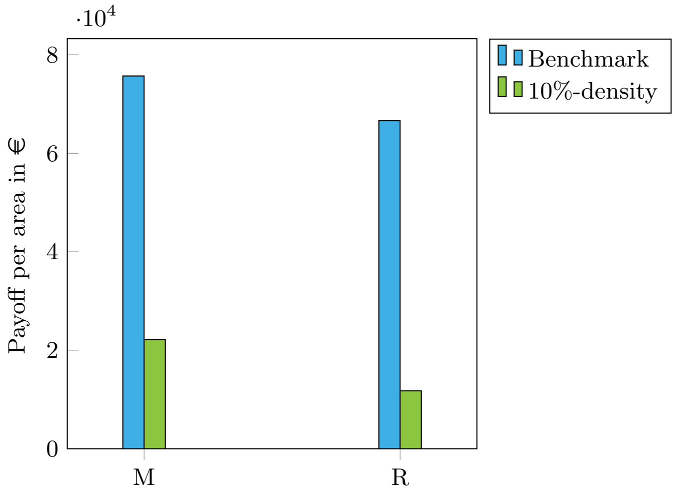
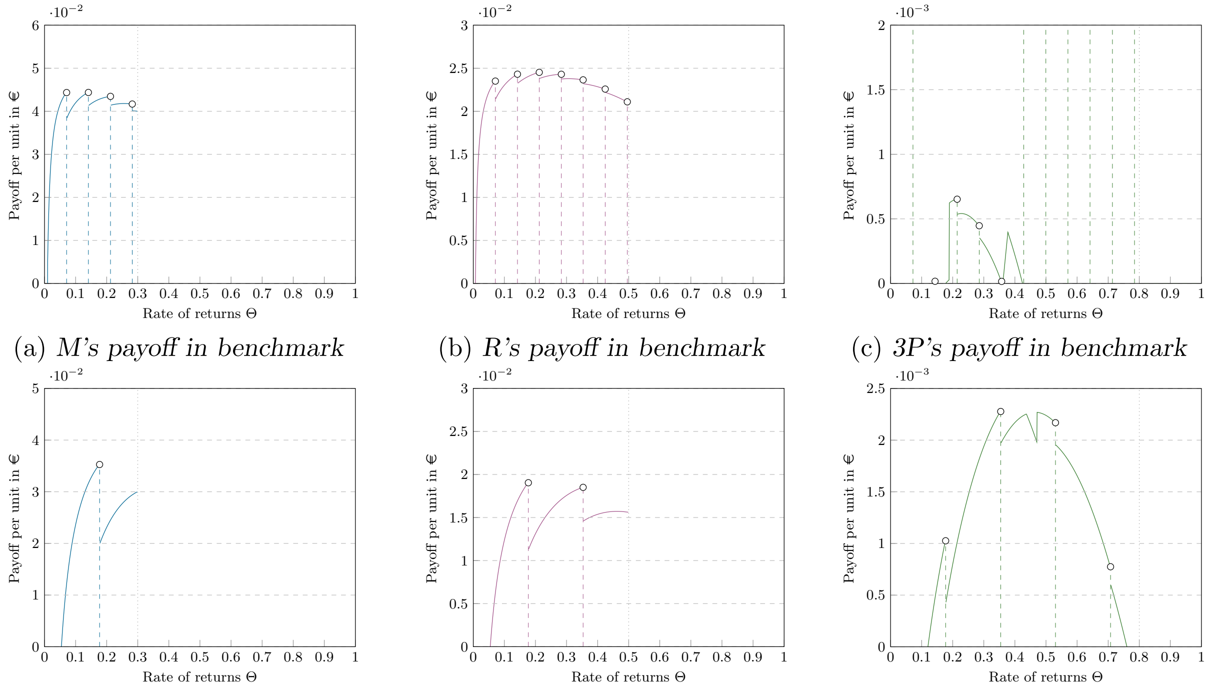


FIGURE 4.9: *The payoff per area with a 10% customer density.*

fuel price of Germany compared to the average fuel price of the market area in the European region of the benchmark scenario is approximated to be 5% higher. Therefore, the milk-run cost is set to €0.00525 and the direct shipment cost to €0.0126 [36]. The land price is estimated to be at approximately the same level as in the benchmark scenario [82]. Nevertheless, the smaller market area results in a lower capacity of the service facilities. The manufacturer has a capacity of 50 000 and the retailer of 12 500. That is why, the cost of operation for the service facilities decrease to €50 000 for the manufacturer and €6 250 for the retailer. The third-parties' logistics network is flexible with changes and thus has a fixed cost of operating a service facility of €1 000 while the capacity is set to 25 000. This information serves as a framework to generate the scenario of a small market area. Comparing the benchmark scenario with the small market area-scenario shows drastic changes in the payoff functions of the players.

In the benchmark scenario in Figure 4.10(a) the manufacturer has to extend the facility four times, whereas within the smaller market area shown in Figure 4.10(d) there are only two extensions necessary, due to a smaller volume of products available for collection. Similar observations can be made comparing the retailer's payoff functions in Figure 4.10(b) and Figure 4.10(e). With the market area size of approximately 282 743 km^2 the retailer has four retail stores available to carry out collections instead of 20 for the market area size of approximately 7 068 583 km^2 in the benchmark scenario. A significant difference shows the payoff function of the third-party. Whereas in the benchmark scenario in Figure 4.10(c) the third-party makes use of 28 facilities, in the scenario with $\ell = 200$ km the third-party occupies only two service facilities to gain positive payoffs.

The maximum payoffs attainable and the rate of returns necessary to reach the optimum are listed in Table 4.11. For the manufacturer and the retailer the maximum payoffs are notably lower with a decrease in market size. In the $\ell = 200$ km-scenario the manufacturer's payoff per unit reduces by approximately 20%. However, the manufacturer's individual collection effort that is necessary to collect the optimum rises by 12%. The rate of collection specific to the retailer drops by 7% and the highest payoff per unit reduces by approximately 22%. In terms of the payoff per area, the highest payoff attainable drops drastically by approximately 98% for the manufacturer with a slightly lower return rate assigned to the payoff and by approximately 97% for the retailer with a decreasing collection rate. On the contrary, the decrease in market area size benefits the third-party as the third-parties' figures already imply. With an increasing



(a) *M's payoff in benchmark* (b) *R's payoff in benchmark* (c) *3P's payoff in benchmark*

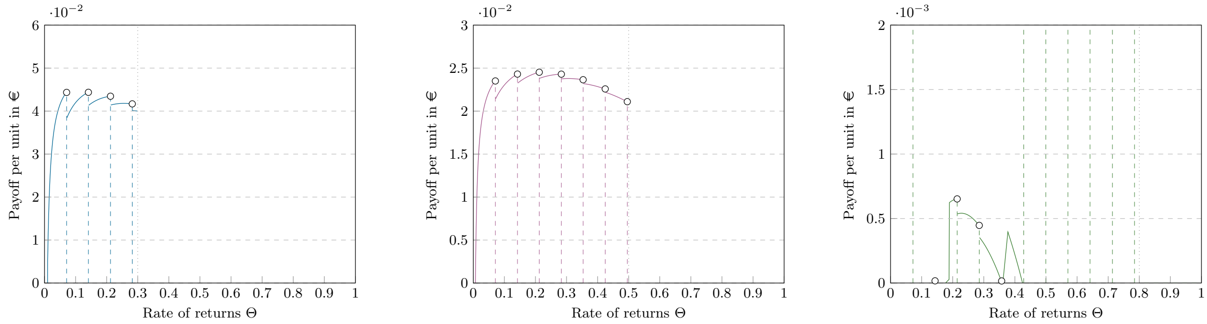
(d) *M's payoff with $\ell = 200$ km* (e) *R's payoff with $\ell = 200$ km* (f) *3P's payoff with $\ell = 200$ km*

FIGURE 4.10: *The effects of a decreasing market area size on the payoff per unit per player.*

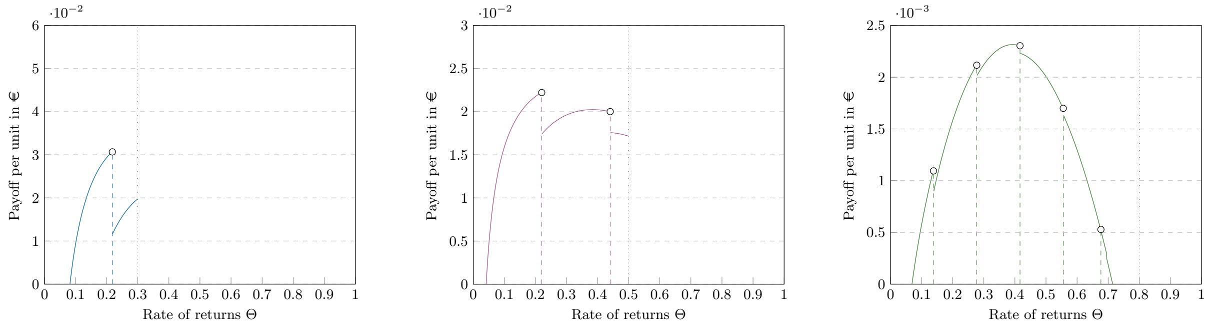
	Benchmark	$\ell = 200$ km
Manufacturer's		
Max. payoff **per unit**	€0.04441	€0.03529
Player specific return rate	47%	59%
Proportion of returns	14%	18%
Max. payoff **per area**	€75 704.15	€1 764.31
Player specific return rate	85%	59%
Proportion of returns	26%	18%
Retailer's		
Max. payoff **per unit**	€0.02453	€0.01906
Player specific return rate	42%	35%
Proportion of returns	21%	18%
Max. payoff **per area**	€66 617.57	€1 851.35
Player specific return rate	85%	71%
Proportion of returns	42%	35%
Third-parties'		
Max. payoff **per unit**	€0.00065	€0.00228
Player specific return rate	27%	44%
Proportion of returns	21%	35%
Max payoff **per area**	€986.21	€278.45
Player specific return rate	27%	55%
Proportion of returns	21%	44%

TABLE 4.11: *The effects of a decreasing market area size on the return rate and payoff per player.*

effort of collection by 17% the highest payoff per unit of the third-party more than triples to the value of €0.00228. Nevertheless, the payoff per area is approximately 72% lower than in the benchmark scenario. The rate of collection increases slightly for the third-party in both cases.



(a) *M's payoff in benchmark* (b) *R's payoff in benchmark* (c) *3P's payoff in benchmark*



(d) *M's payoff with $\ell = 1\,800$ km* (e) *R's payoff with $\ell = 1\,800$ km* (f) *3P's payoff with $\ell = 1\,800$ km*

FIGURE 4.11: *The effects of an increasing market area size on the payoff per unit per player.*

4.2.4 Scenario 4: A large market area

In the fourth scenario, the average distance to the manufacturer is extended to 1 800 km. The original market area grows to a new market area with an extended radius of 2 700 km. An example for this scenario could be a market area in the USA. The fuel price in the USA is notably lower and is approximated to be 50% less than in the European benchmark scenario. This results in €0.0025 for the milk-run and €0.006 for the direct shipping cost [37]. Even though the market area is larger, the land price in the industrial areas are higher [83]. With the larger market area, the capacity of the manufacturer's service facility increases to 5 000 000, the retailer's to 140 000 and the third-parties' to 56 900. The fixed cost of operating a service facility of the third-party is set to €1 000, due to flexibility of location. The increase in capacity together with the higher land price in the industrial areas of the US leads to €5 000 000 for the manufacturer and €34 750 for the retailer for the cost of operating their service facility. The larger market area shows great changes in the level of payoff achievable for each player when compared to the benchmark scenario.

With the larger market area in Figure 4.11(d) the manufacturer has only one interval in which positive payoffs can be achieved, instead of the four in the benchmark scenario. The extended market area has now a size of approximately 22 902 210 km^2 resulting in 36 retail stores that serve as service facilities for the retailer. Comparing the retailer's payoff function per unit in Figure 4.11(b) and Figure 4.11(e), the shape of the function differs, because there are only two positive intervals in the scenario with a large market area. Nevertheless, the drop in payoff per unit is less extreme compared to the scenario that reduces the market area. The third-party has six positive intervals to operate in. This drastic change in the third-parties payoff function per unit is illustrated in Figure 4.11(f). Additionally, the highest payoff per unit of the third-party increases as the third-party makes use of 56 service facilities in the flexible logistics network available to the third-party.

	Benchmark	$\ell = 1\ 800$ **km**
Manufacturer's		
Max. payoff **per unit**	€0.04441	€0.03072
Player specific return rate	47%	73%
Proportion of returns	14%	22%
Max. payoff **per area**	€75 704.15	€153 581.51
Player specific return rate	85%	73%
Proportion of returns	26%	22%
Retailer's		
Max. payoff **per unit**	€0.02453	€0.02224
Player specific return rate	42%	44%
Proportion of returns	21%	22%
Max. payoff **per area**	€66 617.57	€178 061.47
Player specific return rate	85%	77%
Proportion of returns	42%	38%
Third-parties'		
Max. payoff **per unit**	€0.00065	€0.00232
Player specific return rate	27%	49%
Proportion of returns	21%	39%
Max payoff **per area**	€986.21	€20 831.04
Player specific return rate	27%	49%
Proportion of returns	21%	39%

TABLE 4.12: *The effects of an increasing market area size on the return rate and payoff per player.*

In the $\ell = 1\ 800$ km-scenario the manufacturer's highest payoff per unit decreases by approximately 31% while the effort in collection increases up to 73% to reach this payoff. Nevertheless, the payoff per area of the manufacturer rises to €153 581.51 accompanying a collection rate closer to the one of the benchmark scenario. The retailer's payoff per unit is not reduced as drastically as in the $\ell = 200$ km-scenario, but it drops by approximately 9% featuring an only a slight increase in collection rate compared to the benchmark scenario. However, a higher collection effort results in an increased payoff per area of €178 061.47 that is even higher than the manufacturer's payoff per area. At the same time, the third-party benefits greatly from an extending market area size. While the collection effort increases, the maximum payoff per unit achievable rises to €0.00232. Additionally, with an increasing collection rate the payoff per area exceeds the one from the benchmark scenario by approximately 20 times. With the manufacturer collecting 21%, the retailer 38% and the third-party collecting 39% to achieve the highest payoff per area, the limit of 100% collectable units is not exceeded.

4.3 Conclusions and arising questions

This section summarises the conclusions that can be drawn from the scenarios analysed. Section 4.1 and Section 4.2 do not only show interesting conclusions within the scope of investigation, but also give an outlook on further extensions to this study.

The changes in density illustrated in Section 4.1.1 show that with a higher density, the collection effort to reach the highest payoff is lower. Having the highest density by maintaining the best developed logistics network, the third-party benefits from this correlation. Both, manufacturer and retailer could lower the rates of collection significantly, if they where able to make use of a logistics network that could reach more customers through a higher density. In Section 4.1.2 the changes in fuel price directly influencing the transport cost have the strongest influence on the player's highest payoffs. With a rising fuel price, the highest payoff per unit and per area of each player decreases. Especially the third-party is influenced by changes, since the third-party decides on the number of service facilities by balancing out the fixed cost of operating a facility and the transport cost. The changes in fixed cost described in Section 4.1.3 mainly influence the manufacturer and the retailer, since they are using service facilities with predefined locations. Economies of scale are no longer applicable as the number of returns has to be distributed over a higher level of operation cost. Therefore, the higher the land price influencing the cost of operating a service facility, the lower the highest payoff achievable for the manufacturer or the retailer. Only the third-party is effected positively, because the transfer price is directly influenced by the rising reverse logistics cost of the manufacturer. The higher the reverse logistics cost of the manufacturer, the more likely the manufacturer subcontracts the collection task to a partner.

The scenarios with high and low customer density in Section 4.2.1 and Section 4.2.2 show that the customer density does not only influence how much effort each player has to put into the collection of returns, but also emphasis the influence of the customer density on the payoff per area of the players. Therefore, the third-party incorporating the highest customer density, due to the flexible logistics network, has an advantage over the other two players. A smaller or larger market size as described in the scenario analysis of Section 4.2.3 and Section 4.2.4 favours the collection in the third-parties' channel, but also the retailer. This can be an advantage but also a shortcoming of the network flexibility the third-party is able to provide. On the contrary, the service facilities of the retailer and the manufacturer are fixed in number and location and thus it is more difficult for them to balance out economies of scale that cannot be fully obtained. Nevertheless, the retailer benefits from the proximity to the customers.

In general, reasons for these developments can be found in the evolving economies and diseconomies of scale. On the one hand, the third-party profits from synergy effects with other clients. Therefore, with extending market size the third-party can still fully exhibit economies of scale. Especially with a reduction in the size of the market area, this flexibility favours the third-party. On the other hand, with a shirking market area the manufacturer and the retailer are not able to exploit the cost saving potential of economies of scales resulting in a reduction of their profits. Diseconomies of scale originating in the extension of the market area influences their level of payoff per unit negatively. However, economies of scale in the payoff of the overall market area still leads to profits in the manufacturer's and the retailer's channel.

The profits obtained by the manufacturer and the retailer are significantly higher than those of the third-party. However, the fact that third-parties usually have a great variety of clients needs to be taken into account. The third-parties' advantage lies in the possibility to deploy synergy effects between different clients. Therefore, the collection of used products can become a profitable business for a third-party.

The collection of returns cannot only be encouraged by economic motives but also by legislative forces. What would happen if certain levels of the overall collection rate would be demanded by law? Would some form of cooperation between the players evolve to jointly reach the mandated levels? As players would probably not change there "winning strategy" without good reasons, possible tools the legislation could utilise to enforce cooperation amongst the different collecting agents need to be evaluated in addition. The possibility of cooperation between the three collecting agents will be discussed to extend the model of the game.

Most legislations claim responsibility for returns from the manufacturers of the product. Therefore, the possibilities of cooperation are further explored from a manufacturer's point of view. The case in which the manufacturer should carry out the collection as well as the case in which the collection should be subcontracted to either the retailer or the third-party will be examined in detail. Additionally, possible incentives to motivate the collecting partners will be reviewed.

CHAPTER 5

External influences causing cooperation

Contents

In the first extension of the game, the cooperation is caused by legislation introducing laws to enable a certain level of collection. This case will make use of the highest payoff per player in the overall collection area to analyse the situation of conflict.

So far the used products available for collection had been more than enough for each player to collect an optimal number, resulting in the highest payoff for each player. Except for one situation where the return rate was slightly exceeded within a large market area, no cooperation was needed between the three individual decision-makers to reach their highest payoffs as evaluated in Chapter 4. However, what happens if external influences demands the recovery of a certain rate of used products? An example of such a scenario is laws due to the WEEE directive. The waste stream caused by electronic equipment such as mobile phones and computers is one of the fastest growing ones in Europe. The WEEE directive introduces streams for collection and aims at increasing the recycling of the used products [22]. The question arising is whether players are willing to cooperate under the influence of directives demanding a certain collection rate?

5.1 Tools of the legislative

The question a collecting agent is facing when it comes to externally demanded collection rates is whether to cooperate with other players or to work alone. In this game, a player has the options to not cooperate, cooperate with one or cooperate with both players. As there was no motivation to cooperate between the players so far, a threat is introduced by external influences. The legislative wants as many returns as possible to reach environmental goals. For example, the European Commission [21] states in the legislative proposals on waste that it wants to recycle 65% of the municipal waste by 2030. For packaging waste the target is to recycle 75% by 2030. Additionally, the landfill should be reduced to a maximum of 10% of municipal waste. The goal is to incentive manufacturers to bring products on the market that including the opportunity to be recovered or recycled. These goals could be manifested in laws opposed on the different players. In most cases there are certain percentages in collection that a legislative aims for. If these percentages in collection are not met, the legislative places a fine on the responsible collecting agents. Therefore, the influence of two different fine models on cooperation, one incorporating an absolute fine and another one working with a percentage fine, will be tested next.

In the benchmark scenario the player's revenue and cost of the individual payoff functions are listed in Table 5.1. This will be the basis of the numerical example used to analyse the impact of the fine models on cooperation between the players.

	Manufacturer	**Retailer**	**Third-party**
Revenue	€ 100 620.36	€ 100 095.71	€ 4 878.12
Cost	€ 24 916.21	€ 33 478.14	€ 3 891.91
Payoff	€ 75 704.15	€ 66 617.57	€ 986.21

TABLE 5.1: *Revenue, cost and payoff per player in the benchmark scenario.*

5.1.1 Absolute fine

In the first case, absolute fines are imposed on the players. The first stage is a fine of € 900 for not collecting 70% of the returns available. The second stage demands a € 100-fine for not meeting the 90% rate of collection. Aiming for a higher return rate than 90% is not realistic, since all used products would have to find their way back from the customers in the case of a 100% return. The levels of collection are set in alignment with the legislative proposals on waste of the European Commission [21]. The absolute fine is set to a level that still allows the third-party a profitable collection in the benchmark scenario. A sensitivity analysis on the absolute fine is carried out next, to validate this setting.

There are four possible combinations of coalitions described through four different coalition options. The first one is that the manufacturer works alone and the retailer cooperates with the third-party. The manufacturer and the third-party cooperating and collecting against the retailer is a second option. The third option is described by the third-party carrying out the collection alone with the manufacturer and the retailer working together. Finally, all players form the grand coalition working together against an empty coalition. This is the last option and hence number four.

The characteristic function form is deducted for each player to estimate the strength of possible coalitions amongst players. It is set as a lower bound to the payoff each player can reach. If a player would gain less than this value by joining a coalition, the decision-maker should rather play alone. The characteristic function form is determined by the maximin value of each player. The values of the four options are listed in Table 5.2, where $v(\mathcal{P})$ describes the grand coalition and $v(\emptyset)$ the empty coalition. All other combinations are specified by the characteristic function form v including the participants of a coalition in the specific option. Thereby, the combination of M cooperating with R or 3P as well as the cooperation between R and 3P are evaluated.

In the benchmark scenario described in Section 3.4.2, the manufacturer collects 26%, the retailer 42% and the third-party 21% of the amount of used products available for return. Together, they collect 89% of returns in the overall market area. Therefore, if the players want to reach the pre-described rate of 70%, the players have to form a grand coalition. Nevertheless, this will

	Option 1		Option 2		Option 3		Option 4
$v(M)$	€74 804.15	$v(R)$	€65 717.57	$v(3P)$	€86.21	$v(\mathcal{P})$	€143 207.93
$v(R,3P)$	€66 703.78	$v(M,3P)$	€75 790.36	$v(M,R)$	€141 421.71	$v(\emptyset)$	€0.00

TABLE 5.2: *The cooperation options between the three players induced by an absolute fine.*

still not allow them to overstep the 90%-mark. Yet the players are able to distribute the fine of the first stage amongst each other to gain the highest payoff possible under these conditions.

In Section 2.2.2, it is stated that the Core of a game in characteristic function form is defined by all imputations that are not dominated by any other imputation through another coalition. Therefore, the Core of the benchmark scenario is calculated by

$$M \geq €\,74\,804.15, \tag{5.1a}$$
$$R \geq €\,65\,717.57, \tag{5.1b}$$
$$3P \geq €\,86.21, \tag{5.1c}$$
$$M + R \geq €\,141\,421.71, \tag{5.1d}$$
$$M + 3P \geq €\,75\,790.36, \tag{5.1e}$$
$$R + 3P \geq €\,66\,703.78 \text{ and} \tag{5.1f}$$
$$M + R + 3P = €\,143\,207.93. \tag{5.1g}$$

Inserting equation (5.1f) in equation (5.1g) results in

$$M \leq €\,76\,504.15 \tag{5.2}$$

and equation (5.1e) in equation (5.1g) gives

$$R \leq €\,67\,417.57. \tag{5.3}$$

This leads to the Core of the benchmark scenario defined by

$$\begin{aligned} \{(M, R, €\,143\,207.93 - 3P) | €\,74\,804.15 \leq M &\leq €\,76\,504.15 \\ €\,65\,717.57 \leq R \ &\leq €\,67\,417.57\}. \end{aligned} \tag{5.4}$$

The problem of this solution is that the Core has too many imputations to evaluate the best solution. Therefore, the Shapley value of the benchmark scenario is calculated as an alternative solution approach to distribute the payoffs fairly between the participating players of a coalition.

The Shapley value per player is calculated by applying equation (2.6) to the benchmark scenario. Therefore, the manufacturer's Shapley value is given by

$$\begin{aligned} \phi^M &= \frac{1}{3} v(M) + \frac{1}{6}\left(v(M,R) - v(R)\right) + \frac{1}{6}\left(v(M,3P) - v(3P)\right) + \frac{1}{3}\left(v(\mathcal{P}) - v(R,3P)\right) \\ &= €\,75\,670.81. \end{aligned} \tag{5.5}$$

The Shapley value of the retailer is calculated as

$$\begin{aligned} \phi^R &= \frac{1}{3} v(R) + \frac{1}{6}\left(v(M,R) - v(M)\right) + \frac{1}{6}\left(v(R,3P) - v(3P)\right) + \frac{1}{3}\left(v(\mathcal{P}) - v(M,3P)\right) \\ &= €\,66\,584.23. \end{aligned} \tag{5.6}$$

Finally, the third-parties' Shapley value is given by

$$\begin{aligned}\phi^{3P} &= \frac{1}{3}v(3P) + \frac{1}{6}\left(v(M,3P) - v(M)\right) + \frac{1}{6}\left(v(R,3P) - v(R)\right) + \frac{1}{3}\left(v(\mathcal{P}) - v(M,R)\right) \\ &= €\,952.88.\end{aligned} \tag{5.7}$$

The ratio between the Shapley vector and the original payoff per player is compared to decide whether the Shapley value distributes the payoffs fairly amongst the participating players of a coalition with the help of

$$M = \frac{€\,75\ 670.81}{€\,75\ 704.15} = 0.9996 \qquad R = \frac{€\,66\ 584.23}{€\,66\ 617.57} = 0.9995 \qquad 3P = \frac{€\,952.88}{€\,986.21} = 0.9662. \tag{5.8}$$

The comparison proves a fair distribution of payoffs as all ratios are closely together. In general, all players attain a higher payoff from cooperating with their opponents in the benchmark scenario with fines introduced by external influences.

What would happen, if only two of the players cooperate, even though it would not allow them to overstep the predefined collection rate of 70%? Would the distribution of the fine as well as joint payoffs be preferable to a cooperation in the grand coalition? The two possible choices each player can make is either to cooperate with one of the opponents (Option 2 (a) or 2 (b)) or to carry out the collection alone (Option 1). All options are described by M, R or 3P and are listed in Table 5.3.

	Manufacturer		Retailer		Third-Party
Option 1					
$v(M)$	€ 74 804.15	$v(R)$	€ 65 717.57	$v(3P)$	€ 86.21
Option 2					
(a) $v(M,R)$	€ 141 421.71	**(a)** $v(R,M)$	€ 141 421.71	**(a)** $v(3P,M)$	€ 75 790.36
(b) $v(M,3P)$	€ 75 790.36	**(b)** $v(R,3P)$	€ 67 234.78	**(b)** $v(3P,R)$	€ 67 234.78

TABLE 5.3: *The cooperation options between two players induced by an absolute fine.*

The manufacturer and the retailer cooperating in the benchmark scenario with a perspective of the entire market area as in Section 3.4.2 will reach 68%. The manufacturer and the third-party cooperating will collect up to 47% of the used products available in the market area. Additionally, a cooperation between the third-party and the retailer will collect 63% of the returns. The fine of € 900 is imposed on the players, since the lower limit of 70% is not exceeded. However, the retailer can encourage the third-party to collect up to 29% by splitting the additional cost of collection of € 269. Thereby, the retailer and the third-party collect 71% collectively and thus only have to pay the € 100-fine. This results in the retailer and the third-party paying € 369 for their coalition. Splitting extra cost is not a feasible option for the manufacturer. Additionally, it is not possible to split the cost between the retailer and the third-party in the three-player coalition, since it would lower the manufacturer's payoff.

Depending on the choice of coalition option, the Core of the players is either determined by

$$M \geq €\,74\ 804.15, \tag{5.9a}$$

$$R \geq €\,65\ 717.57, \tag{5.9b}$$

$$M + R = €\,141\ 421.71, \tag{5.9c}$$

or by

$$M \geq €\,74\,804.15, \tag{5.10a}$$
$$3P \geq €\,86.21, \tag{5.10b}$$
$$M + 3P = €\,75\,790.36, \tag{5.10c}$$

or, finally, by

$$R \geq €\,65\,717.57, \tag{5.11a}$$
$$3P \geq €\,86.21, \tag{5.11b}$$
$$R + 3P = €\,67\,234.78. \tag{5.11c}$$

With the help of equation (5.9a) – (5.9c) when cooperating with the retailer or by equation (5.10a) – (5.10c) when cooperating with the third-party, the manufacturer's Core would be denoted by $\{€\,74\,804.15 \leq M \leq €\,75\,704.15\}$. The Core of the retailer when working together with the manufacturer described by equation (5.9a) – (5.9c) would be determined by $\{€\,65\,717.57 \leq R \leq €\,66\,617.56\}$, while working together with the third-party as in equation (5.11a) – (5.11c) would lead to the retailer's Core of $\{€\,65\,717.57 \leq R \leq €\,67\,148.57\}$. Additionally, the third-parties' Core that can be attained through cooperation with the manufacturer is defined by $\{€\,86.21 \leq 3P \leq €\,986.21\}$ and with the retailer described by $\{€\,86.21 \leq 3P \leq €\,1\,517.21\}$. The upper bound of each player's Core represents the highest payoff per player in the original scenario. As in the benchmark scenario with cooperation between three players, the Core of each player has many imputations. The Shapley values for each player will be evaluated in addition, since it is difficult to select one imputation from the Core.

The Shapley value of the manufacturer cooperating with either the retailer or the third-party is given by

$$\begin{aligned} \phi_{2p}^{M} &= \frac{1}{2}v(M) + \frac{1}{2}\left(v(M,R) - v(R)\right) \text{ or} \\ &= \frac{1}{2}v(M) + \frac{1}{2}\left(v(M,3P) - v(3P)\right) \\ &= €\,75\,254.15. \end{aligned} \tag{5.12}$$

Cooperating with the manufacturer, the retailer's Shapley value is calculated by

$$\begin{aligned} \phi_{2p}^{R} &= \frac{1}{2}v(R) + \frac{1}{2}\left(v(R,M) - v(M)\right) \\ &= €\,66\,167.57, \end{aligned} \tag{5.13}$$

whereas cooperating with the third-party, the Shapley value of the retailer is given by

$$\begin{aligned} \phi_{2p}^{R} &= \frac{1}{2}v(R) + \frac{1}{2}\left(v(R,3P) - v(3P)\right) \\ &= €\,66\,433.07. \end{aligned} \tag{5.14}$$

Finally, the third-parties' Shapley value cooperating with the manufacturer is calculated as

$$\begin{aligned} \phi_{2p}^{3P} &= \frac{1}{2}v(3P) + \frac{1}{2}\left(v(3P,M) - v(M)\right) \\ &= €\,536.21, \end{aligned} \tag{5.15}$$

while the cooperation with the retailer leads to the Shapley value of the third-party given by

$$\begin{aligned} \phi_{2p}^{3P} &= \frac{1}{2}v(3P) + \frac{1}{2}\left(v(3P,R) - v(R)\right) \\ &= €\,801.71. \end{aligned} \tag{5.16}$$

Comparing the Shapley values of two-player-cooperations obtained in equation (5.12) for the manufacturer, equations (5.13) and (5.14) for the retailer and equations (5.15) and (5.16) for the third-party to the Shapley values of the cooperation amongst all three players in equation (5.5) – (5.7), the three-player cooperations reach higher levels of profit than the two-player ones. Therefore, each player attains the highest payoff that is comparable to the highest payoff per area of the benchmark scenario in the non-cooperative game within the three-player cooperation. That is why, the cooperation of all three players will be analysed further.

The sensitivity of the introduced fines for not reaching a certain level of collection rate are analysed in the following. The size of the fines for not meeting certain collection rates were chosen by setting fines to levels that do not exceed the highest payoff of the third-party with the lowest payoff amongst all players. This assumption ensures that the collection of returns stays profitable for all players. First of all, the absolute fine is increased by 1%, then by 5% and finally by 10% to analyse changes in the model. This will result in fines higher than the profit of the third-party. Afterwards, the fines will be lowered by 1%, 5% and 10% to investigate the effects that decreasing fines have on the formation of coalitions and the payoffs of their members.

A rise by 1% would cause the fines to increase to €909 for not reaching a 70% collection rate and to €101 for not reaching the 90%-level. Besides, the fines would increase to €945 and €990 for less than 70% in the 5%- and 10%-scenario respectively. For less than 90% of the overall collection rate in the 5%-scenario the fines would increase to €105 and in the 10%-scenario to €110. For the three-player cooperation, Table 5.4 shows the Shapley value per player. Only the highest Shapley value is displayed for the two-player cooperation. The manufacturer, the retailer and the third-party get a lower payoff with an increase in fines. With an increase of 10%, all player's payoffs decrease by €3.33 in comparison to the benchmark. A similar phenomenon can be observed in the scenario where two players cooperate. The manufacturer that is directly influenced by the increase in fine looses €45, while the retailer's and the third-parties' Shapley value decreases by €5 in the +10%-example. Additionally, the characteristic function form of the third-party reveals that collection is no longer profitable for a non-cooperating third-party if the fine is increasing.

	Benchmark	Fine +1%	Fine +5%	Fine +10%	Fine -1%	Fine -5%	Fine -10%
3-player							
M	75 670.81	75 670.48	75 669.15	75 667.48	75 671.15	75 672.48	75 674.15
R	66 584.23	66 583.90	66 582.57	66 580.90	66 584.57	66 585.90	66 587.57
3P	952.88	952.55	951.21	949.55	953.22	954.55	956.21
2-player							
M	75 254.15	75 249.65	75 231.65	75 209.15	75 258.65	75 276.65	75 299.15
R	66 433.07	66 432.57	66 430.57	66 428.07	66 433.57	66 435.57	66 438.07
3P	801.71	801.21	799.21	796.71	802.21	804.21	806.71
1-player							
M	74 804.15	74 795.15	74 759.15	74 714.15	74 813.15	74 849.15	74 894.15
R	65 717.57	65 708.57	65 672.57	65 627.57	65 726.57	65 762.57	65 807.57
3P	86.21	77.21	41.21	−3.79	95.21	131.21	176.21

TABLE 5.4: *The sensitivity analysis on the absolute fines in €.*

Table 5.4 does not only illustrate what happens if fines increase, but also the effect of a decrease in the absolute fine. Similar percentages are used in this extension. The absolute fine dropping by 10% results in €810 in the first stage and leads to €90 in the second stage of the fine. In addition, a change of 5% leads to €855 as well as €95 for the first and the second stage of a decreasing absolute fine. The 1% decrease results in a €891- and €99-fine. The effects of a drop in fines is mirrored to an increase in fines. The payoff per player increases by €3.34 for each player that cooperates in the three-player cooperation in the 10%-scenario. If cooperation between two players is taking place, the manufacturer's payoff is €45 and the retailer's and the third-parties' payoff is €5 higher than in the 10%-scenario. Additionally, the collection of the third-party stays profitable with and without cooperation. In conclusion, a valid level of absolute fine is necessary for the simulation of the model as it influences the payoff of each player directly. In some cases, the absolute fine can determine if a player has to cooperate to keep the collection profitable.

5.1.2 Percentage fine

What would happen if the fine was not absolute but a certain percentage of the players' payoffs? Especially comparing the influence of the absolute fine on the manufacturer and retailer to the third-party, reveals that the absolute fine has the greatest impact on the third-party. The overall payoff from collection is not as great as the payoff of the manufacturer and the retailer, since the third-party has the opportunity to work with different clients. Nevertheless, imposing a fine that is a certain percentage of the player's payoff might be a fairer way to encourage cooperation through external influences. Would the likeliness of cooperation increase even further? The second fine model imposes a 50%-fine of the player's payoff for not reaching 70% of collection rate and 30% for not obtaining 90% of the returns of all used products. This system is comparable to tax, for example, where a certain percentage gets subtracted from an employee's salary. The different stages of the fine are comparable to the absolute fine model, while the setting of the percentage levels is validated in a sensitivity analysis.

The options of cooperation are similar to the system with absolute fines. The players have four options, listed in Table 5.5, to cooperate with one opponent or form the grand coalition $v(\mathcal{P})$ that confronts the empty coalition $v(\emptyset)$. In general, the strength of the coalitions defined by their characteristic function forms are weaker when compared to the absolute fine system. Only the third-party benefits as the fine deducted is smaller in this case. For the manufacturer and the retailer the fines payable are higher. Especially the characteristic function form of the grand coalition drops by €42 892.38.

	Option 1		Option 2		Option 3		Option 4
$v(M)$	€37 852.07	$v(R)$	€33 308.78	$v(3P)$	€493.11	$v(\mathcal{P})$	€100 315.55
$v(R,3P)$	€33 801.89	$v(M,3P)$	€38 345.18	$v(M,R)$	€71 160.86	$v(\emptyset)$	€0.00

Table 5.5: *The cooperation options between three players induced by a percentage fine.*

The Core of a game in the benchmark scenario with the percentage fine model is calculated by

$$M \geq €\,37\ 852.07, \tag{5.17a}$$
$$R \geq €\,33\ 308.78, \tag{5.17b}$$
$$3P \geq €\,493.11, \tag{5.17c}$$
$$M + R \geq €\,71\ 160.86, \tag{5.17d}$$
$$M + 3P \geq €\,38\ 345.18, \tag{5.17e}$$
$$R + 3P \geq €\,33\ 801.89 \text{ and} \tag{5.17f}$$
$$M + R + 3P = €\,100\ 315.55. \tag{5.17g}$$

Equation (5.17f) inserted in equation (5.17g) gives

$$M \leq €\,66\ 513.66 \tag{5.18}$$

and equation (5.17e) substituted in equation (5.17g) results in

$$R \leq €\,61\;970.37. \tag{5.19}$$

This leads to the Core of the benchmark scenario defined by

$$\begin{aligned}\{(M, R, €\,100\;315.55 - 3P) | €\,37\;852.07 \leq M \leq €\,66\;513.66 \\ €\,33\;308.78 \leq R \;\leq €\,61\;970.37\}.\end{aligned} \tag{5.20}$$

The Shapley value will be deducted for the case with the percentage fines, since the Core did not assign distinct results to the different cooperation options. The manufacturer's Shapley value is calculated in the same way as the one with absolute fines. Therefore, it is given by

$$\begin{aligned}\phi^M &= \frac{1}{3}v(M) + \frac{1}{6}\left(v(M,R) - v(R)\right) + \frac{1}{6}\left(v(M,3P) - v(3P)\right) + \frac{1}{3}\left(v(\mathcal{P}) - v(R,3P)\right) \\ &= €\,47\;405.94.\end{aligned} \tag{5.21}$$

The retailer's Shapley value is calculated similarly and follows as

$$\begin{aligned}\phi^R &= \frac{1}{3}v(R) + \frac{1}{6}\left(v(M,R) - v(M)\right) + \frac{1}{6}\left(v(R,3P) - v(3P)\right) + \frac{1}{3}\left(v(\mathcal{P}) - v(M,3P)\right) \\ &= €\,42\;862.65.\end{aligned} \tag{5.22}$$

Additionally, the Shapley value of the third-party is given by

$$\begin{aligned}\phi^{3P} &= \frac{1}{3}v(3P) + \frac{1}{6}\left(v(M,3P) - v(M)\right) + \frac{1}{6}\left(v(R,3P) - v(R)\right) + \frac{1}{3}\left(v(\mathcal{P}) - v(M,R)\right) \\ &= €\,10\;046.97.\end{aligned} \tag{5.23}$$

Except for the third-parties', all Shapley values are lower then in the previous scenario. Therefore, the question arises whether the distribution of the payoffs amongst the players is fairer or not. This question is addressed by comparing the payoff of each player without cooperation to the Shapley value per player by

$$M = \frac{€\,47\;405.94}{€\,75\;704.15} = 0.6262 \quad R = \frac{€\,42\;862.65}{€\,66\;617.57} = 0.6434 \quad 3P = \frac{€\,10\;046.97}{€\,986.21} = 10.1875. \tag{5.24}$$

It is clear from expression (5.24) that the Shapley value of the percentage fine does not distribute payoffs fairer than the absolute fine. The third-party benefits more from a cooperation than in the absolute fine model.

For the manufacturer, the retailer and the third-party it is more profitable to cooperate in the grand coalition than to play alone. Cooperation between two players will be investigated with percentage fines for all possible cooperation options to deepen the conclusions. The constellation of cooperation is similar to the model with absolute fines. The players can either play alone (Option 1) or cooperate with the first or second opponent (Option 2(a) or 2(b)) as described in Table 5.6, while the retailer and the third-party split the cost of the third-parties' additional collection effort.

Therefore, the Shapley value of the manufacturer cooperating with either the retailer or the third-party is deducted by

$$\begin{aligned}\phi^M_{2p} &= \frac{1}{2}v(M) + \frac{1}{2}\left(v(M,R) - v(R)\right) \text{ or} \\ &= \frac{1}{2}v(M) + \frac{1}{2}\left(v(M,3P) - v(3P)\right) \\ &= €\,37\;852.07.\end{aligned} \tag{5.25}$$

	Manufacturer		Retailer		Third-Party
Option 1					
$v(M)$	€ 37 852.07	$v(R)$	€ 33 308.78	$v(3P)$	€ 493.11
Option 2					
(a) $v(M,R)$	€ 71 160.86	**(a)** $v(R,M)$	€ 71 160.86	**(a)** $v(3P,M)$	€ 38 345.18
(b) $v(M,3P)$	€ 38 345.18	**(b)** $v(R,3P)$	€ 47 134.35	**(b)** $v(3P,R)$	€ 47 134.35

TABLE 5.6: *The cooperation options between two players induced by a percentage fine.*

The retailer's Shapley value cooperating with the manufacturer is calculated as

$$\begin{aligned}\phi_{2p}^{R} &= \frac{1}{2}v(R) + \frac{1}{2}\left(v(R,M) - v(M)\right)\\ &= €\,33\,308.78,\end{aligned} \tag{5.26}$$

while the Shapley value of the cooperation with the third-party is given by

$$\begin{aligned}\phi_{2p}^{R} &= \frac{1}{2}v(R) + \frac{1}{2}\left(v(R,3P) - v(3P)\right)\\ &= €\,39\,975.01.\end{aligned} \tag{5.27}$$

The Shapley value of the third-party that cooperates with the manufacturer is calculated as

$$\begin{aligned}\phi_{2p}^{3P} &= \frac{1}{2}v(3P) + \frac{1}{2}\left(v(3P,M) - v(M)\right) \text{ or}\\ &= \frac{1}{2}v(3P) + \frac{1}{2}\left(v(3P,R) - v(R)\right)\\ &= €\,493.11,\end{aligned} \tag{5.28}$$

while the third-parties' Shapley value of the cooperation with the retailer is deducted by

$$\begin{aligned}\phi_{2p}^{3P} &= \frac{1}{2}v(3P) + \frac{1}{2}\left(v(3P,R) - v(R)\right)\\ &= €\,7\,159.33.\end{aligned} \tag{5.29}$$

In all three cases of cooperation with the manufacturer, the cooperation amongst two players gives the same result as for each player carrying out the collection alone. This is a special case of the percentage fine model. The equal distribution of the fine does not benefit cooperation in a two-player scenario. Only if the retailer and the third-party split the extra cost of overcoming the 70%-fine, the value of the two-player cooperation differs.

The sensitivity of the percentage fine level is analysed with the same scenarios of 1%, 5% and 10% increase and decrease on the rate of the fine as in the sensitivity analysis of the absolute fine model. A 1% increase of the 50%-fine for not reaching 70% collection rate leads to a 50.5%-fine, while the 30%-fine for not reaching 90% changes to a 30.5%-fine. An increase of 5% results in a fine of 52.5% and of 31.5%. Additionally, rising the fine by 5% leads to the first stage of 55% and the second stage of 33% of the fine model. A 49.5%-fine and a 29.7%-fine, on the other hand, results from a 1 % decrease as well as a 5% drop leads to a 47.5%-fine as well as a 28.5%-fine. The decrease by 10% leads to fines of 45% and 27%. Displayed in Table 5.7 are the results of this sensitivity analysis.

In the example with a 10% increase in the three-player cooperation, the manufacturer's payoff decreases by € 2 829.83 and the retailer's payoff decreases by € 2 375.5. However, the third-parties' payoff increases by € 906.07. The 1% and 5% rise show the same developments but

	Benchmark	Fee +1%	Fee +5%	Fee +10%	Fee -1%	Fee -5%	Fee -10%
3-player							
M	47 405.94	47 122.95	45 991.02	44 576.11	47 688.92	48 820.85	50 235.76
R	42 862.65	42 625.10	41 674.90	40 487.15	43 100.20	44 050.39	45 238.14
3P	10 046.97	10 137.58	10 500.01	10 953.04	9 956.36	9 593.93	9 140.89
2-player							
M	37 852.07	37 473.55	35 959.47	34 066.87	38 230.59	39 744.68	41 637.28
R	39 975.01	39 709.93	38 649.61	37 324.21	40 240.09	41 300.41	42 625.82
3P	7 159.33	7 222.41	7 474.72	7 790.10	7 096.26	6 843.95	6 528.57
1-player							
M	37 852.07	37 473.55	35 959.47	34 066.87	38 230.59	39 744.68	41 637.28
R	33 308.78	32 975.70	31 643.34	29 977.91	33 641.87	34 974.22	36 639.66
3P	493.11	488.17	468.45	505.42	498.04	517.76	542.42

TABLE 5.7: *The sensitivity analysis on the percentage fines in €.*

with smaller values. The payoff decrease of the third-party is contrary to the developments in the absolute fine model. With two-player cooperation, the manufacturer loses €1 892.6 with the 5% increase and further €1 892.6 with a change of 10%. A rise of 5% leads to a drop of €1 325.4 in the payoff of the retailer and further €1 325.4 with 10%. Nevertheless, the third-parties' payoff increases by €315.39 for a 5% and further €315.38 for a 10% change. The conclusions of the two-player cooperation are consistent with those of the absolute fine model. The third-party benefits from cooperating with the other players. With an increase in fine, the chances of cooperation increase and so does the third-parties' payoff.

In the case of the percentage fines the decrease is consistent with the model with absolute fines. The three-player cooperation leads to an increase in payoff for the manufacturer by €2 829.92 for a drop of 10% and by €1 414.91 with a 5% decrease. The retailer also shows an increase in payoff by €2 375.49 for a 10% and by €1 187.74 for a 5% change. Only the third-party experiences a decrease of €906.08 for the 10%- and a drop by €453.04 in the 5%-scenario. Therefore, the results of the increase in percentage fines are mirrored. With cooperation between two players, the results are increasing showing similarities to the model with the absolute fine. A 10% drop leads to an increase by €3 785.21 for the manufacturer's payoff. The retailer increases by €2 650.81 and the third-parties' payoff decreases by €630.76 in the -10%-case. Additionally, the 1% and 5% decrease reveals a similar development but less significant changes in values.

Comparing the absolute fine and the percentage fine model reveals that the absolute fine is more suitable to encourage cooperation amongst players. The percentage fine model already divides the fine fairly among the participating agents. Therefore, players can gain less from cooperation. Even though the amount of fines received is lower with the absolute fine, the primary goal to collect as many returns a possible is enticed better with the help of the absolute fine model.

5.2 Sensitivity and scenarios of cooperation

The absolute fine has been chosen as an efficient tool to motivate players for cooperation amongst each other. In addition, the cooperation between three players forming a grand coalition $v(\mathcal{P})$ proved to be the most beneficial form of coalition to all players. In Chapter 4 the environment in which the collection takes place has been changed in the parameters of density, variable and fixed cost. Additionally, the four different scenarios with a high customer density, a low customer density, a small market area and a large market area have been analysed. The Shapley values will be calculated for three- and two-player cooperations as well as for each player playing alone and compared to the benchmark scenario to evaluate if these changes affect the formation of coalitions.

5.2.1 Sensitivity of the cooperations

The stability of the cooperations will be tested first. What will happen, if the customer density of each player, the fuel price or the land price of either the manufacturer or the retailer will be changed by ±1%, ±5% and ±10%? Does the formation of cooperation between the players stay the same as in the benchmark scenario?

For all players, cooperating in the grand coalition results in the highest payoffs per area as illustrated in Table 5.8. For the manufacturer and the retailer the payoff per area stays the same for small changes in the customer density. However, changing the customer density does affect the payoff of the third-party. An increase in customer density decreases the payoff per area of the third-party obtained through the grand coalition. An increase in density by 10% results in the manufacturer, the retailer and the third-party exceeding the second stage of the absolute fine. Additionally, the manufacturer and retailer as well as the third-party and retailer exceed the first stage of the fine. Therefore, they can gain a higher payoff from the two-player cooperation than from the grand coalition. The changes in customer density result in the third-party having to cooperate to keep collections profitable. Trying to overcome the fine of not collecting 90% in the case of −10% for example is not profitable, as the fine is smaller than the burden of collecting more products that would have to be carried by the partners in the grand coalition. In conclusion, the observations resemble the results obtained in Section 4.1.1.

	Bench.	**Den. +1%**	**Den. +5%**	**Den. +10%**	**Den. -1%**	**Den. -5%**	**Den. -10%**
3-player							
M	75 670.81	75 670.81	75 704.15	80 092.32	75 670.81	65 158.01	44 370.36
R	66 584.23	66 584.23	66 617.67	72 996.31	57 972.13	57 972.13	51 505.20
3P	952.88	916.38	822.43	539.18	981.34	1 012.83	1 553.22
2-player							
M+R	75 254.15	75 254.14	75 254.15	80 175.66	75 254.15	64 741.34	43 953.69
M+3P	75 254.15	75 254.14	75 254.15	79 775.66	75 254.15	64 741.34	43 953.69
R+M	66 167.57	66 167.57	66 167.57	72 679.65	57 555.46	57 555.46	51 088.53
R+3P	66 433.07	66 527.24	66 356.94	72 679.65	57 924.41	57 735.00	51 088.53
3P+M	536.21	499.71	372.43	222.51	564.67	596.17	1 136.55
3P+R	801.71	859.39	561.81	622.51	933.62	775.70	1 136.55
1-player							
M	74 804.15	74 804.14	74 804.15	79 325.66	74 804.15	64 291.34	43 503.69
R	65 717.57	65 717.57	65 717.57	71 829.65	57 105.46	57 105.46	50 638.53
3P	86.21	49.71	−77.57	−227.49	114.67	146.17	686.55

TABLE 5.8: *The sensitivity analysis on changes in density with absolute fines in €.*

With a change in transport cost due to an increasing fuel price, the manufacturer and the retailer still collect as many returns as both players collect in the benchmark scenario. The third-party increases the rate of collection slightly with a rising fuel price. Therefore, a cooperation in the grand coalition allows the players to cross the 90%-limit. Nevertheless, with an increase in transport cost, the Shapley value of all players decrease in every possible coalition option listed in Table 5.9. Additionally, the third-party needs to cooperate to keep the collection economic. A decrease in fuel price mirrors these results, thus leading to an increase in payoff for every player compared to the benchmark scenario. All three collecting agents achieve the highest Shapley value while operating in the grand coalition. Sharing the burden of exceeding the second stage fine is profitable for the retailer and the third-party in some cases of the two-player coalition.

Generally, the third-party is the collecting agent that is effected the most by changes in fuel price. Thereby, the observation show consistency with the results from Section 4.1.2.

	Bench.	**Fuel +1%**	**Fuel +5%**	**Fuel +10%**	**Fuel -1%**	**Fuel -5%**	**Fuel -10%**
3-player							
M	75 670.81	75 614.38	75 422.50	75 142.12	75 727.29	75 953.74	76 271.30
R	66 584.23	66 542.74	66 410.07	66 202.57	66 625.73	66 791.73	67 032.56
3P	952.88	935.17	892.42	795.88	960.57	1 028.36	1 140.64
2-player							
M+R	75 254.15	75 197.71	74 972.50	74 692.12	75 310.63	75 537.08	75 821.30
M+3P	75 254.15	75 197.71	74 972.50	74 692.12	75 310.63	75 537.08	75 821.30
R+M	66 167.57	66 126.07	65 960.07	65 752.57	66 209.07	66 375.06	66 582.56
R+3P	66 433.07	66 387.29	65 960.07	65 795.74	66 524.00	66 706.13	66 582.56
3P+M	536.21	518.50	442.42	277.21	543.91	611.69	690.64
3P+R	801.71	779.73	442.42	677.21	858.84	942.76	690.64
1-player							
M	74 804.15	74 747.71	74 522.50	74 242.12	74 860.63	75 087.08	75 371.30
R	65 717.57	65 676.07	65 510.07	65 302.57	65 759.07	65 925.06	66 132.56
3P	86.21	68.50	−7.58	−104.12	93.91	161.69	240.64

TABLE 5.9: *The sensitivity analysis on changes in fuel price with absolute fines in €.*

The fixed cost is changed by increasing and decreasing the cost of operating the manufacturer's and the retailer's facilities. In the case of an increase in facility cost of the manufacturer, the payoff of the manufacturer is directly influenced and the payoff of the third-party indirectly. The most profitable option of the manufacturer stays the collection as part of the grand coalition, even though the Shapley value is higher with an increasing land price as illustrated in Table 5.10. The manufacturer collects more returns, comparable to the observations in Chapter 4.1.3. With a decrease in land price, the payoff of the manufacturer decreases as the collection effort of the manufacturer is lower. The retailer is indirectly influenced through splitting the cost of additional collection with the third-party in the two-player coalition. Nevertheless, for both players the grand coalition stays the most profitable option.

	Bench.	**M +1%**	**M +5%**	**M +10%**	**M −1%**	**M −5%**	**M −10%**
3-player							
M	75 670.81	75 756.74	76 084.00	76 457.93	75 583.19	75 215.19	74 713.52
R	66 584.23	66 584.23	66 584.23	66 584.23	66 584.23	66 584.23	66 584.23
3P	952.88	970.83	1 042.65	1 132.42	934.92	863.11	773.33
2-player							
M+R	75 254.15	75 340.07	75 667.33	76 041.26	75 166.52	74 798.52	74 296.85
M+3P	75 254.15	75 340.07	75 667.33	76 041.26	75 166.52	74 798.52	74 296.85
R+M	66 167.57	66 167.57	66 167.57	66 167.57	66 167.57	66 167.57	66 167.57
R+3P	66 433.07	66 436.06	66 448.03	66 463.00	66 430.07	66 518.85	66 237.06
3P+M	536.21	554.17	625.98	715.76	518.26	715.76	356.67
3P+R	801.71	822.66	906.45	1 011.19	780.76	797.72	426.16
1-player							
M	74 804.15	74 890.07	75 217.33	75 591.26	74 716.52	74 348.52	73 846.85
R	65 717.57	65 717.57	65 717.57	65 717.57	65 717.57	65 717.57	65 717.57
3P	86.21	104.17	175.98	265.76	68.26	−3.56	−93.33

TABLE 5.10: *The sensitivity analysis on changes in M's land price absolute fines in €.*

If the operating cost of the retailer increases, only the retailer is influenced directly. The results are listed in Table 5.11. Additionally, all collection rates stay the same. The grand coalition is the most profitable option again, even though the retailer looses some of the highest payoff. With a decrease in fixed price, the highest payoff of the retailer increases and vice versa.

	Bench.	**R** +1%	**R** +5%	**R** +10%	**R** −1%	**R** −5%	**R** −10%
3-player							
M	75 670.81	75 670.81	75 670.81	75 670.81	75 670.81	75 670.81	75 670.81
R	66 584.23	66 506.67	66 198.20	65 813.04	66 660.91	66 969.39	67 335.43
3P	952.88	952.88	952.88	952.88	952.88	952.88	952.88
2-player							
M+R	75 254.15	75 254.15	75 254.15	75 254.15	75 254.15	75 254.15	75 254.15
M+3P	75 254.15	75 254.15	75 254.15	75 254.15	75 254.15	75 254.15	75 254.15
R+M	66 167.57	66 090.01	65 781.53	65 396.37	66 244.25	66 552.72	66 938.76
R+3P	66 433.07	66 355.51	66 047.03	65 661.87	66 509.75	66 818.22	67 204.26
3P+M	536.21	536.21	536.21	536.21	536.21	536.21	536.21
3P+R	801.71	801.71	801.71	801.71	801.71	801.71	801.71
1-player							
M	74 804.15	74 804.15	74 804.15	74 804.15	74 804.15	74 804.15	74 804.15
R	65 717.57	65 640.01	65 331.53	64 946.37	65 794.25	66 102.72	66 488.76
3P	86.21	86.21	86.21	86.21	86.21	86.21	86.21

TABLE 5.11: *The sensitivity analysis on changes in R's land price absolute fines in €.*

In conclusion, all results of the sensitivity analysis on the cooperation reflect the conclusions from the non-cooperative game. Splitting the burden of a higher collection to get around the second stage fine is profitable in most cases for the retailer and the third-party cooperation.

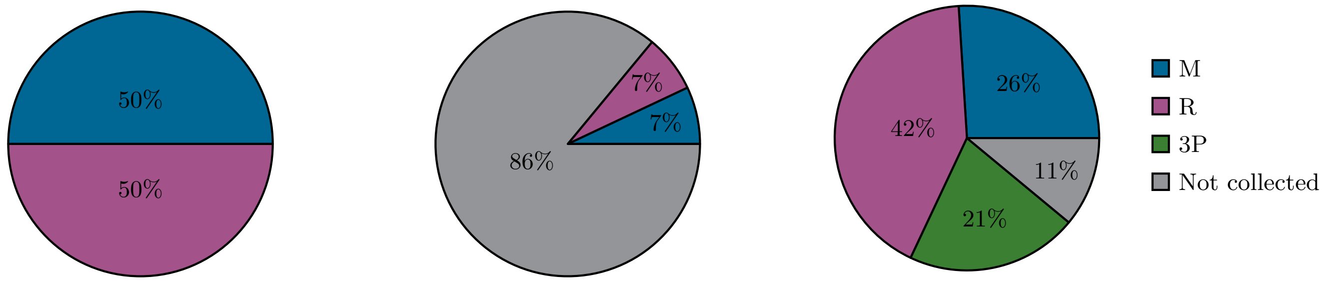
5.2.2 Cooperation in different scenarios

Secondly, the evaluation of the cooperations in the four different scenarios will be addressed. Will the cooperations stay the same in the scenario with a high and a low customer density? How will cooperations form if the market is located in a small country in Europe or in the large market area of the USA? In Table 5.12 the four scenarios are compared to the benchmark scenario.

	Benchmark	**Scenario 1**	**Scenario 2**	**Scenario 3**	**Scenario 4**
3-player					
M	75 670.81	–	–	1 464.31	153 314.84
R	66 584.23	–	–	1 951.35	178 194.81
3P	952.88	–	–	378.45	20 964.38
2-player					
M+R	75 254.15	106 705.44	21 744.21	1 314.31	153 131.51
M+3P	75 254.15	–	–	1 314.31	153 131.51
R+M	66 167.57	72 729.65	11 316.01	1 401.35	177 611.47
R+3P	66 433.07	–	–	1 801.35	178 011.47
3P+M	536.21	–	–	−171.55	20 381.04
3P+R	801.71	–	–	228.45	20 781.04
1-player					
M	74 804.15	105 805.44	21 294.21	864.31	152 681.51
R	65 717.57	71 829.65	10 866.01	951.35	177 161.47
3P	86.21	–	–	−621.55	19 931.04

TABLE 5.12: *The scenario analysis on changes with absolute fines in €.*

In the first two scenarios, the density of customers, that is influenced by the logistics network of the collecting agent individual to each player, is changed. There are two different cases that will be provided next. The density is set to 100% for every player in the first case. Compared to the benchmark scenario, the manufacturer and the retailer collect 50%. There are no returns



(a) 100%-*density* (b) 10%-*density* (c) *Benchmark*

FIGURE 5.1: *The proportion of returns per player with changing customer density.*

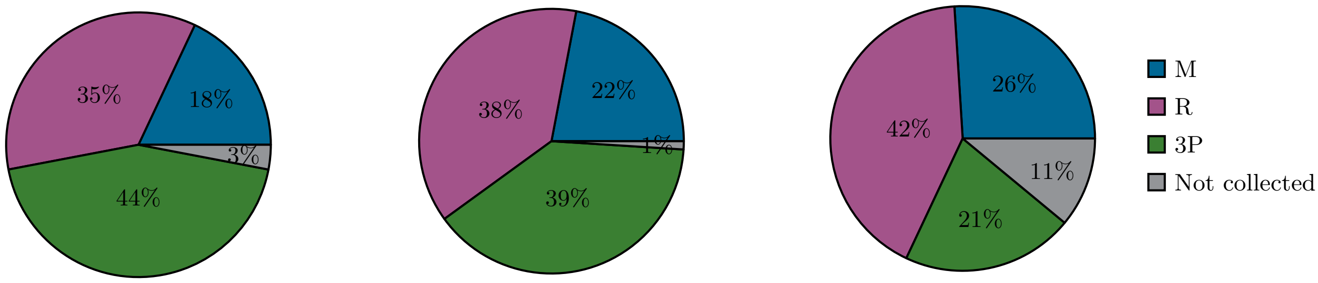
available for the third-party. As depicted in Figure 5.1, the retailer and the manufacturer jointly collecting overstep the 70%- and the 90%-collection limit and thus only have to pay no fine in case of cooperation. The Shapley values of the two-player coalition are higher than the characteristic function forms of each player. The collection effort decreases with an increase in customer density for the manufacturer and the retailer.

In the second case that incorporates a 10% customer density, the manufacturer and the retailer have very low levels of collection rates as illustrated in Figure 5.1. The third-party does not collect at all, since even with cooperation it is not profitable for this player. The Shapley values attained by cooperating in the two-player coalition between the manufacturer and the retailer are significantly lower than in the benchmark scenario. However, the two-player coalitions leads to higher results attainable than with each player carrying out the collection alone.

In the third and the fourth scenario, the market area size is changed. First the market is located in a small area and then extended to a larger size. The market area of Scenario 3 has a size of 300 km with a change in ℓ to 200 km. The manufacturer and the retailer lower their collection effort to 18% and 35%, while the third-party increases the effort in collection to collect 44% of the returns available in the market area. The 70%-limit is exceeded by a cooperation between the retailer and the third-party. All collection rates added up result in exceeding the second stage of the fine model of 90%. The collection rates of the three players are depicted in Figure 5.2. Nevertheless, the manufacturer, the retailer and the third-party have significantly lower characteristic function forms than in the original scenario. Especially for the third-party and also for the manufacturer collection of returns is not profitable any more without cooperative collection. The manufacturer's, the retailer's and the third-parties' Shapley values decrease. The grand coalition stays the most beneficial choice for all players.

An alternative to a shirking market area is proposed with the extended market area of Scenario 4 with a radius of 2700 km by changing ℓ to 1 800 km. Within the extended market area, the manufacturer decreases collection compared to the benchmark scenario at a rate of 22% and the retailer at 38%. The third-parties' collection is the most profitable with a rate of 39%. This is illustrated in Figure 5.2. Only the cooperation between retailer and the third-party does exceed the lower collection limit in this case. All players experience an increase in Shapley value with the extending market area. Therefore, the three-player cooperation is the most profitable for the manufacturer, the retailer and the third-party.

Changing scenarios and not only single parameters has great effects on the formation of a coalitions. Additionally, in some scenarios, the collection is not profitable for the third-party even with the possibility to cooperate.



(a) ℓ = *200 km* (b) ℓ = *1 800 km* (c) *Benchmark*

FIGURE 5.2: *The proportion of returns per player with changing market size.*

5.3 Conclusions on the cooperative case

A fine for not reaching a certain collection level is an effective tool of the legislative to motivate individual decision-makers to cooperate and jointly collect the returns. A realistic way to impose a fine on the collecting agents is to introduce a two-stage fine model. The first limit is set to a level that can easily be achieved, sometimes even with two-player cooperation. The level of the second limit is rather high and claims a collection of almost all used products available for return. This second stage should not be set too high and only serve as a motivation for players to collect even more returns by splitting the burden of the extra collection effort. The application of the Shapley value gives a distinct and fair distribution of the fine and jointly achieved payoff amongst all cooperation partners.

The comparison in Section 5.1 between an absolute and a percentage fine imposed on the collecting agents if the predefined limits are not achieved, favours the introduction of an absolute fine model. An absolute fine encourages the cooperation between the players to share the fine and distribute the payoffs fairly amongst each other. The percentage fine demands a payment that is already matchable to each player's individual payoff. By analysing the level of the fine it is observable that an increase and a decrease of the fine shows mirrored results in the Shapley value. Therefore, the Shapley value distributes the fines and payoffs evenly amongst the members of a coalition. Nevertheless, setting up a valid fine is crucial to the model.

The benchmark scenario reveals that the higher the absolute fine, the more the cooperation amongst all three players becomes the most profitable option. In addition, cooperation between two players is less likely to occur and no cooperation leaves each decision-maker with the lowest payoff. In a three-player cooperation each player is able to obtain almost as much as if no fine would be demanded.

In Section 5.2 the behaviour of the players in situations in which the customer density, the variable and the fix cost are changing are evaluated. The results of this evaluation are consistent with the conclusions from the non-cooperative version of the game. Additionally, Section 5.2.2 describes the influence of different scenarios on the formation of coalitions. The parameters of the scenario are important to the collection rates and payoffs that can be obtained by each player in a coalition.

The collection in the grand coalition is the most profitable option to all players. If legislative imposes a fine for not collecting certain levels of returns, the grand coalition is the way to obtain the highest payoff. Therefore, cooperation between individual decision-makers is an important consideration to make reverse logistics profitable.

CHAPTER 6

A change in perspective causing cooperation

Contents

In this chapter, the point of view will be changed to that of the manufacturer to evaluate which channel is the most profitable to carry out the collection for the manufacturer. The situation of conflict is analysed by evaluating the highest payoff per player in the overall collection area.

All collecting agents had been modelled as individual decision-makers up to this point. Therefore, the point of view equals the collection agents. However, legislative often places the responsibility for recovery processes into the hands of the manufacturer. For example, the WEEE directive tries to establish manufacturer responsibility [20]. It encourages the design and manufacturing of electrical and electronic equipment that takes repair, upgrade, re-use or recycling of used products into account. The recycled material should be integrated into new products. At least the collection of the returns should be financed by the manufacturers, to take on the responsibility for the waste from their products. That is why, it is time to change the point of view from each individual player to the manufacturer's perspective. Is it possible to develop a form of cooperation where the manufacturer, that is still responsible for the forward logistics, let another collecting agent carry out the collection in the reverse channel? How do the payoff functions need to change to allow a cooperation within the closed-loop supply chain? Additionally, what would be an optimal channel choice of the manufacturer in the different scenarios? The following sections will address these questions.

6.1 Payoff functions from the manufacturer's perspective

First of all, using the retailer's or the third-parties' collection channel, the manufacturer's payoff functions need to be re-defined. Afterwards, the payoff functions of the retailer and the third-party are developed further to allow cooperation between the manufacturer and the collecting partner.

6.1.1 The manufacturer using the retailer's channel

In the collection channel of the retailer, the manufacturer's objective function ${}^{R}\Pi_M$ according to Savaskan et al. [73] is given by

$$\max\ {}^{R}\Pi_M = (\phi - \beta p)[w - c_m + \Theta_R \Delta] - b_R \Theta_R (\phi - \beta p)\ . \tag{6.1}$$

Applying $b_R = \Delta$ due to a collection contract between the manufacturer and the retailer, the manufacturer's profits increase constantly. A change in the wholesale price w or the cost of re-manufacturing c_m could influence the manufacturer's profit per unit differently. However, with an optimal wholesale price w the manufacturer's profit, according to Savaskan et al. [73], is calculated as

$${}^{R}\Pi_M = \frac{(\phi - \beta\, c_m)^2/(8\beta)}{1 - \beta(\Delta - {}^{M}C_D)(b_R - {}^{M}C_D)/(4R_M)}\ . \tag{6.2}$$

In the benchmark scenario, the profit per unit converges to approximately €0.0515 for the manufacturer's highest payoff. The retailer's objective function ${}^{R}\Pi$ can be calculated by applying equation (3.31). This leads to similar results as in Section 3.4.1.

6.1.2 The manufacturer using the third-parties' channel

The manufacturer's objective function in the third-parties' collection channel ${}^{3P}\Pi_M$, according to Savaskan et al. [73], is given by

$$\max\ {}^{3P}\Pi_M = (\phi - \beta p)[w - c_m + \Theta_{3P}(\Delta - b_{3P})]\ . \tag{6.3}$$

A collection contract between the third-party and the manufacturer leads to the application of equation (3.34) as defined in Section 3.3. Therefore, the cost is incorporated directly. If the transfer price equals the savings per unit from re-manufacturing, the profit per unit is only influenced by the recovery cost and the wholesale price. According to Savaskan et al. [73] the profit of the manufacturer with an optimal wholesale price w is given by

$${}^{3P}\Pi_M = \frac{(\phi - \beta\, c_m)^2/(8\beta)}{1 - \beta(\Delta - b_{3P})(b_{3P} - {}^{M}C_D)/(4R_M)}\ . \tag{6.4}$$

The third-parties' payoff is given by equation (3.33). Therefore, the profit per unit is at a constant level of approximately €0.0511 in the benchmark scenario. The third-parties' payoff is given by equation (3.33). This payoff function leads to results as in Section 3.4.1, revealing the third-parties' highest payoff with the corresponding return rate.

6.2 The manufacturer playing in the benchmark scenario

If the manufacturer's profit per unit increases or stays the same in the benchmark scenario, it is most likely that the highest payoff will be achieved within the channel that is able to collect the most returns. However, the retailer as well as the third-party have an optimal rate of collection. An optimal rate of collection is computed per area. In the collection channel of the retailer, an optimal collection rate per area of the retailer would result in a payoff per area of €154 664.44 for the manufacturer. This resembles the maximum volume of returns the retailer can collect within the benchmark scenario. The third-parties' optimal collection rate per area would result in €77 239.87 payoff per area for the manufacturer. Nevertheless, extending the collection up to the third-parties' limit of possible collections, the manufacturer could obtain up to €265 455.39. That is why, a one-time payment could be paid to the operator of each channel to encourage the collecting agent to carry out further collections. This one-time payment could be seen as an incentive to collect even beyond an optimal collection rate of the business partner's channel, yet keeping the collection profitable to the operator. In the benchmark scenario, the case of the third-party could result in a higher payoff for the manufacturer when subcontracting the collection.

6.2.1 The arbitration procedure between the manufacturer and the retailer

In the benchmark scenario, the retailer's highest payoff per area is €66 617.57 that is achieved by collecting 42% of the returns available in the overall market area. This is the highest level of collection the retailer can carry out. The manufacturer would gain €154 664.44 when subcontracting the collection to the retailer. The new profit is €78 960.29 higher than collecting within the manufacturer's own channel.

For the manufacturer, collection in the manufacturer's channel or in the retailer's channel is possible. However, from the perspective of the retailer collecting in the manufacturer's channel is not a realistic option. The payoff resembles the manufacturer raising the collection up to the maximum, while the retailer gaining payoff from the maximum collection rate. The manufacturer and the retailer jointly collecting, enables the retailer to keep collecting the retailer's optimal payoff. The situation is illustrated in the following table.

		R	
		Channel M	Channel R
M	Channel M	(€75 704.15, €0.00)	(€75 704.15, €66 617.57)
	Channel R	(€154 664.44, €66 617.57)	(€154 664.44, €66 617.57)

The procedure to find the arbitration pair for the two-player cooperation between manufacturer and retailer, as described in Section 2.2.2, illustrates the payoff region in a Cartesian coordinate system as in Figure 6.1. The maximin value of the manufacturer and the retailer are given by €154 664.44 and €66 617.57 respectively. Both maximin values are incorporated in Figure 6.1. All Pareto-optimal points that lie north-east of the maximin values form the bargaining set. In this case, the bargaining set is the arbitration pair at point (€154 664.44, €66 617.57). Furthermore, the manufacturer and the retailer are able to negotiate over the level of a side-payment to promote future business relations for example.

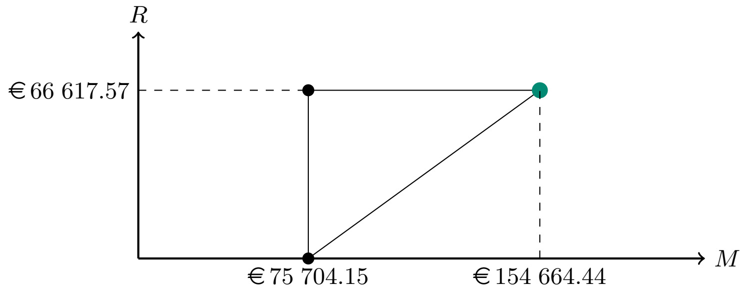


FIGURE 6.1: *The region of cooperation between M and R.*

6.2.2 The arbitration procedure between M and 3P

The highest payoff of the third-party is € 986.21, that is achieved by collecting 21% of the used products in the market area. The third-party would loose € 22 282.39 by extending the collection, thus making the collection up to the third-parties' limit of 73% highly unlikely. The manufacturer would have to pay € 22 538.96 to keep the third-parties' collection profitable. This would still leave the manufacturer with a profit of € 243 173.00 that is € 167 468.85 higher than the collection in the manufacturer's channel.

The manufacturer chooses whether to use the own collection channel or subcontract the third-party. As the third-party has no choice of collecting within the manufacturer's structures, the third-party in the manufacturer's channel describes what happens without cooperation between the players. The normal collection effort is illustrated in the second option as the third-parties' collection channel. In the following table the situation between the manufacturer and the third-party is displayed.

		3P	
		Channel M	Channel 3P
M	Channel M	(€ 75 704.15, € 0.00)	(€ 75 704.15, € 986.21)
	Channel 3P	(€ 265 455.39, € −22 282.39)	(€ 243 173.00, € 986.21)

The arbitration procedure between the manufacturer and the third-party results in the payoff region in Figure 6.2.

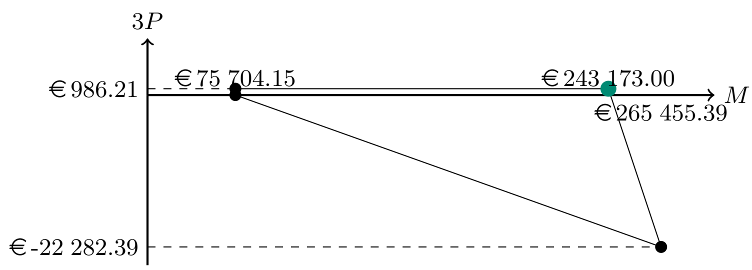


FIGURE 6.2: *The region of cooperation between M and 3P.*

The maximin values for the manufacturer and the third-party are calculated in the next step. The manufacturer's payoff matrix is depicted as

(€ 75 704.15)	(€ 75 704.15)
(€ 265 455.39)	(€ 243 173.00)

with Column 2 dominating Column 1. Therefore, the matrix of the manufacturer is reduced to

(€ 75 704.15)
(€ 243 173.00)

,

which leaves Row 1 being dominated by Row 2 and results in the manufacturer's maximin value of € 243 173.00. Additionally, the matrix of the third-party is illustrated by

(€ 0.00)	(€ −22 282.39)
(€ 986.21)	(€ 986.21)

.

Row 2 dominates Row 1 in this table, reducing the matrix to

(€ 986.21)	(€ 986.21)

.

Therefore, the arbitration pair of the cooperation between manufacturer and third-party is at (€ 243 173.00, € 986.21) again leaving room for future negotiations. This analysis investigates on the cooperation possibilities arising from the benchmark scenario.

6.3 Different games from the manufacturer's point of view

How would the possibilities of cooperation change, if the environment in which the collection takes place changes? Does a change in one of the three parameters density, variable or fixed cost benefit or harm the collecting agents? How would a change in the scenario affect the different channels? An evaluation on these influences will focus on the manufacturer, as the payoffs for the collection partners stay the same as in Chapter 4 due to the retailer's and the third-parties' maximin values.

6.3.1 The influence of single parameter changes

With a changing customer density the payoff of the manufacturer stays at a constant level until the size of the facility is changed when collecting within the own channel. The results of this analysis are described in Table 6.1. If the customer density increases, the payoff achievable by the retailer is higher. If the density is decreased, the retailer's payoff decreases accordingly. A subcontract with the third-party leads to higher results with an increase in density, since the manufacturer carries the loss from the third-parties' channel while more returns are collected. The third-parties' channel collects the most returns, thus the highest profits are obtained from subcontracting the third-party.

	Bench.	**Den. +1%**	**Den. +5%**	**Den. +10%**	**Den. -1%**	**Den. -5%**	**Den. -10%**
Manu.							
in M	75 704.15	75 704.14	75 704.15	80 225.66	75 704.15	65 191.34	44 403.69
in R	154 664.44	154 691.23	154 783.90	212 559.55	129 111.94	129 998.32	111 380.09
in 3P	243 173.00	259 070.97	252 499.45	247 326.97	241 021.16	239 859.56	231 859.75

TABLE 6.1: *The sensitivity analysis on changes in density from M's point of view in €.*

Changes in the fuel price leading to variations in transport cost describes the second part of the analysis resulting Table 6.2. The manufacturer carrying out the collection reduces the payoff slowly while the fuel price rises. A decrease in fuel price leads to an increase of the manufacturer's payoff. Subcontracting the retailer or third-party shows similar results on the effect of fuel price as for the manufacturer collecting the own channel. The third-parties channel seems to be the best option for the manufacturer. Though highly dependent on the transport cost, the manufacturer obtains high payoffs while using the third-parties' channel compared to the other channel options.

	Bench.	**Fuel +1%**	**Fuel +5%**	**Fuel +10%**	**Fuel -1%**	**Fuel -5%**	**Fuel -10%**
Manu.							
in M	75 704.15	75 647.71	75 422.50	75 142.12	75 760.63	75 987.08	76 271.30
in R	154 664.44	154 658.88	154 636.74	154 609.31	154 670.01	154 692.39	154 720.60
in 3P	243 173.00	243 036.14	242 590.47	241 901.16	243 378.81	243 925.59	244 526.44

TABLE 6.2: *The sensitivity analysis on changes in fuel price from M's point of view in €.*

The changes in fixed cost caused by changing land prices of the manufacturer influences the manufacturer's, the retailer's and the third-parties' channel, while changing the land prices of the retailer only influences the retailer's collection channel. Nevertheless, operating in the third-parties' channel leads to a similar payoff level as achieved in the benchmark scenario. Therefore, subcontracting the third-party stays the most beneficial option of the manufacturer.

With an increase in land price of the manufacturer, subcontracting the third-party obtains higher payoffs as the fixed cost of the manufacturer increases. A decreasing land price leads to a decrease in the third-parties' payoffs as well as a decrease in the manufacturer's attainable payoffs, since the manufacturer lowers the collection effort. The increase in land price of the manufacturer results in an increase in payoff per area of the retailer and vice versa. Nevertheless, subcontracting the third-party leads to higher payoffs per area in every scenario than any other collection option.

	Bench.	**M +1%**	**M +5%**	**M +10%**	**M -1%**	**M -5%**	**M -10%**
Manu.							
in M	75 704.15	75 790.07	76 117.33	76 491.26	75 616.52	75 248.52	74 746.85
in R	154 664.44	154 640.83	154 548.28	154 436.81	154 688.23	154 785.29	154 910.88
in 3P	243 173.00	243 304.36	243 830.49	244 489.66	243 041.70	242 517.17	241 863.01

TABLE 6.3: *The sensitivity analysis on changes in M's land price from M's point of view in €.*

The manufacturer's and the third-parties' channels are not influenced by the change in fixed cost of the retailer. Therefore, the manufacturer obtains the same payoffs as in the benchmark scenario. Additionally, the changes in fixed cost of the retailer are too small to influence the payoff per area if subcontracted by the manufacturer. Only the retailer experiences a decrease or increase with changing land price within the own channel.

	Bench.	**R +1%**	**R +5%**	**R +10%**	**R -1%**	**R -5%**	**R -10%**
Manu.							
in R	154 664.44	154 664.44	154 664.44	154 664.44	154 664.44	154 664.44	154 664.44

TABLE 6.4: *The sensitivity analysis on changes in R's land price from M's point of view in €.*

The observations from the sensitivity analysis of the different channels through the manufacturer's perspective are consistent with the analysis in the non-cooperative and cooperative version of the game. Nevertheless, a change in a single parameter does influence the highest payoff attainable for the manufacturer in every channel.

6.3.2 The influence of changing scenarios

Four different scenarios showing the effects of a high customer density, a low customer density, a small market area and a large market area on the channel choice of the manufacturer. The results are listed in Table 6.5.

	Bench.	**Scenario 1**	**Scenario 2**	**Scenario 3**	**Scenario 4**
Manu.					
in M	75 704.15	106 705.44	22 194.21	1 764.31	153 581.51
in R	154 664.44	180 029.52	25 837.89	5 155.74	471 640.94
in 3P	243 173.00	–	35 415.89	10 243.68	841 268.94

TABLE 6.5: *The scenario analysis from M's point of view in €.*

Scenario 1 with a customer density of 100% for every collecting agent emphasises the importance of this parameter. If all collecting agents would collect with the same density, the retailer would be the best choice for the manufacturer. Compared to the manufacturer's channel or the third-parties' channel, the manufacturer is able to obtain the highest profit with subcontracting the retailer, because of the higher market power. In Scenario 2 the customer density is reduced to a minimum of 10%. Although the manufacturer has to compensate for the third-parties' losses the collection via the third-party is the most profitable, due to low transport cost. Collecting through the retailer is the best option. The small market area of Scenario 3 benefits the retailer's collection channel. Nevertheless, the highest payoff is attained by the third-party. In the fourth scenario, the market area is extended. This development is most beneficial to the third-parties' channel.

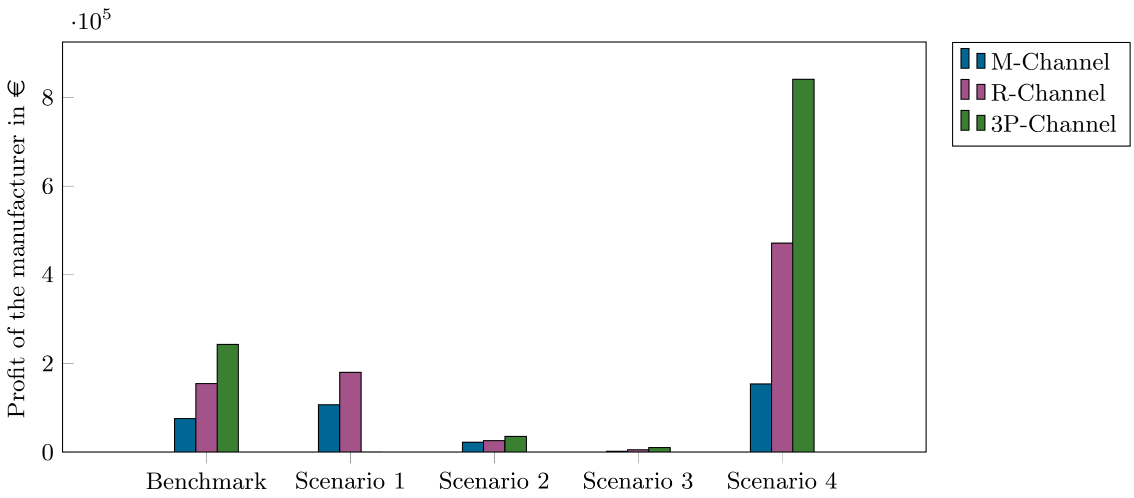


FIGURE 6.3: *The manufacturer's profit in different scenarios.*

The different scenarios show that the channel of the retailer can be the best choice if all players have the same customer density. Nevertheless, the third-party is beneficial in most scenarios, as pointed out by the sensitivity analysis and the benchmark scenario, since it can collect the most returns due to the highest customer density. These observations are illustrated in Figure 6.3.

6.4 Conclusions on the manufacturer's point of view

In the benchmark scenario, the best choice for the manufacturer is to subcontract the collection effort to the third-party. The third-party has the lowest logistics cost as well as the best collection network. This is achieved through high flexibility of the network structure and synergy effects with other clients. With support of the manufacturer through one-time payments, the third-party is able to collect the highest rate of returns available on the market and still gain the highest profit achievable, if collection was carried out alone. These conclusions are stable as small changes in single parameters does not influence the choice of channel of the manufacturer.

However, depending on the scenario in which the collection takes place, the favour can turn towards the retailer. If all players have the same level of density, the manufacturer achieves the highest profits by subcontracting to the retailer, since the profit that the manufacturer gains through the retailer's channel is increasing. Especially, when the existing networks become more important due to the proximity to the customer, the retailer is the best option. In changing market sizes, on the other hand, the third-party is the best collection partner. The third-party can deal better with the requirements of a changing market sizes, due to the flexibility of the third-parties' network. Additionally, the third-party with the highest customer density is able to collect the highest level of returns.

The introduction of a fine by the legislative seems less probable with the manufacturer subcontracting a collection partner. The collection rates of the individual decision-makers fluctuate around 20% and 40%. With the manufacturer subcontracting the third-party in most scenarios, the collection rate of 80% gets satisfied automatically.

Cooperation in the form of the manufacturer subcontracting the third-party and the retailer might occur, when the legislative demands up to 90% of the used products to be returned. In a cooperation between the retailer's and the third-parties' channel, the retailer could collect the returns in the smaller areas around the retail stores and the third-party could carry out the collection of the used products that are further away from the manufacturer's recovery facility. It is most likely that the manufacturer would accept a fine and still gain more than collecting within the own channel. However, the fine might increase in this case.

In conclusion, an optimal collection channel from the manufacturer's point of view has been evaluated as an alternative form of cooperation if legislative calls the manufacturer to take responsibility of the end-of-life products.

CHAPTER 7

Conclusion

Contents

A summary of the content of the thesis is provided in this chapter. Additionally, the achievement of the six objectives is pointed out and the contributions of the thesis are stated. The conclusions drawn in this chapter have three different aspects. The non-cooperative and the cooperative version of the game that had been modelled and analysed in Chapter 3 and Chapter 5 are compared briefly first. Secondly, the conclusions of this project and the current state of scientific knowledge extracted from the papers in Section 2.3 will be aligned with the outcome of the analysis with the changing perception in Chapter 6. The question of choosing an optimal reverse channel structure are discussed for the benchmark scenario as the third aspect. Finally, ideas for future research are given to complete the thesis.

7.1 Thesis summary

The concepts of forward and reverse logistics are introduced in the beginning of the thesis. Afterwards, the two-fold motivation – economic reasons and legislative regulations – are described. The problem of how to choose an optimal reverse channel structure to collect used products is identified as part of Chapter 1.

The basic understanding of the problem and the current state of scientific knowledge is provided in Chapter 2. Therefore, a literature review is carried out. Not only general information on CLSCs and game theory are given, but also the combination of the two topics is pointed out.

Reverse logistics is modelled using a game theory approach in Chapter 3. Therefore, the different reverse channel structures form the different players of the game. The competition between these players is illustrated by the simulation of the non-cooperative version of the game at the end of Chapter 3.

The stability of solutions is tested by changing single parameters at a time in the sensitivity analysis of the game in Chapter 4. Additionally, different market scenarios in which the game could possibly take place are investigated.

The thesis is further extended to not only provide insights into the non-cooperative, but also into the cooperative version of the game. Therefore, cooperation is caused by external influences in Chapter 5 and by a change in perspective to the manufacturer's point of view in Chapter 6. The stability of these versions as well as the influence of different market scenarios is analysed.

In Chapter 7 the non-cooperative and the cooperative version of the game are contrasted. Additionally, the observations of the thesis are compared to the current state of scientific knowledge that was obtained in the literature review. The thesis is completed by an outlook on opportunities for future studies on this topic.

7.2 The achievement of objectives

The following objectives were pursued throughout the thesis towards the aim of choosing an optimal reverse channel.

OBJECTIVE I: Perform a literature review on the current body of scientific knowledge.

OBJECTIVE II: Model different reverse logistic options with a game theory approach.

OBJECTIVE III: Test the stability of the modelled game.

OBJECTIVE IV: Investigate different scenarios and versions of the modelled game.

OBJECTIVE V: Interpret the results.

OBJECTIVE VI: Draw conclusions on the choice of an optimal reverse logistics structure.

In fulfilment of Objective I, a literature review on the topics of closed-loop supply chains and game theory was carried out in the second chapter. Thereby, papers that combine reverse logistics with game theory approaches were investigated in particular. Objective II was achieved by modelling the different reverse channel options and their specific networks as players of the non-cooperative and cooperative version of a game. In fulfilment of Objective III and IV a sensitivity and a scenario analysis was carried out. The stability of the solution of the game subject to the parameters was investigated by changing one of the three most influential parameters of transport cost – the customer density, the fuel price and the land price – at a time. Additionally, the game was played in four different scenarios with a high customer density, a low customer density, a small and a large market area. All results were interpreted in achievement of Objective V. Finally, Objective VI is fulfilled by applying the conclusions of this thesis to determine an optimal reverse channel structure in a benchmark scenario.

7.3 Thesis contributions

Two different comparisons, one between non-cooperative and cooperative games and the other one between the state of the current scientific knowledge and the conclusions of this thesis are presented. Additionally, the question of an optimal reverse channel structure will be discussed for the benchmark scenario.

7.3.1 Non-cooperative versus cooperative games

A comparison between the non-cooperative and the cooperative game is given, since these two versions of the game are established in the thesis. The stability of the versions is evaluated. Additionally, the versions will not only be compared in the benchmark scenario, but also in other scenarios.

The economic optimality for the individual collecting agent is defined by the motivation to engage into the reverse logistics business. If there was no prescribed collection rate, an optimal choice of the players in the benchmark scenario would be to carry out the collection alone to obtain the highest profit achievable in the market area. However, if legislation imposes a fine on not reaching a certain collection rate serving as the second aspect of motivation, each collecting agent could gain a higher payoff by operating in the grand coalition than carrying out the collection alone as illustrated in Figure 7.1. Through cooperation, the manufacturer, the retailer and the third-party would only get €33.33 less than without external fines. Therefore, the grand coalition would be the best choice for each player in the benchmark scenario with fines imposed by legislation.

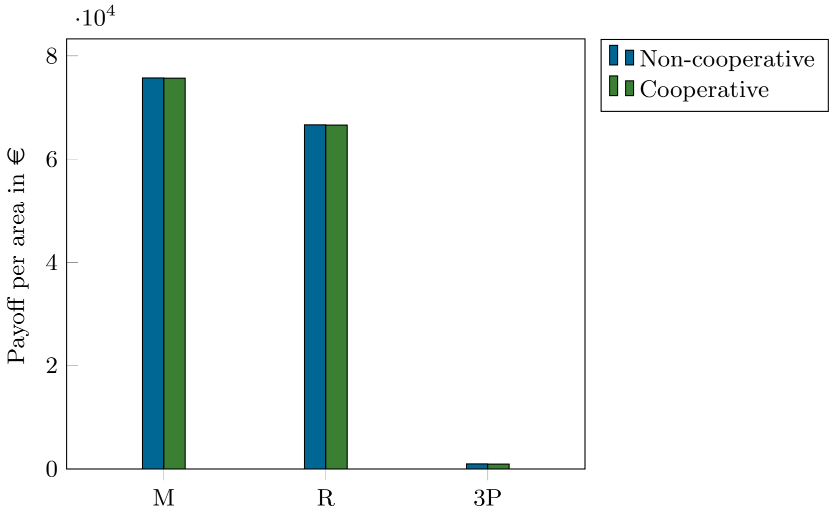


FIGURE 7.1: *The difference in payoff per area between the non-cooperative and cooperative version of the game in the benchmark scenario with the introduction of absolute fines.*

Both versions of the game have similar stabilities. A change in customer density influences the individual collection rates of the manufacturer and the retailer in the non-cooperative and cooperative game. Nevertheless, the third-party is influenced significantly by a change in density as the logistics network is designed flexible to customer density, transport and fixed cost. In the non-cooperative version of the game, a change in fuel price leading to a change in transport cost causes a decrease in payoff for the manufacturer, the retailer and the third-party accompanied by a constant collection rate. The version of the game that is cooperative, shows similar results. For each player, operating in the grand coalition is the most profitable option as the increase in cost of transport can be balanced out by all players collectively. For the third-party it is important to cooperate in this scenario to keep collection profitable. If the land price of the manufacturer increases, the manufacturer's payoffs increase due to higher collection rates. Nevertheless, the third-parties' payoffs increase with the manufacturer's land price as the transfer price rises. If the retailer's land price increases, on the other hand, only the affected retailer's channel becomes less profitable than in the benchmark scenario of the non-cooperative game. Therefore, the affected player's try to encourage the other players to cooperate in the grand coalition. The grand coalition is likely to form, since it stays the most profitable option for every player in the cooperative version.

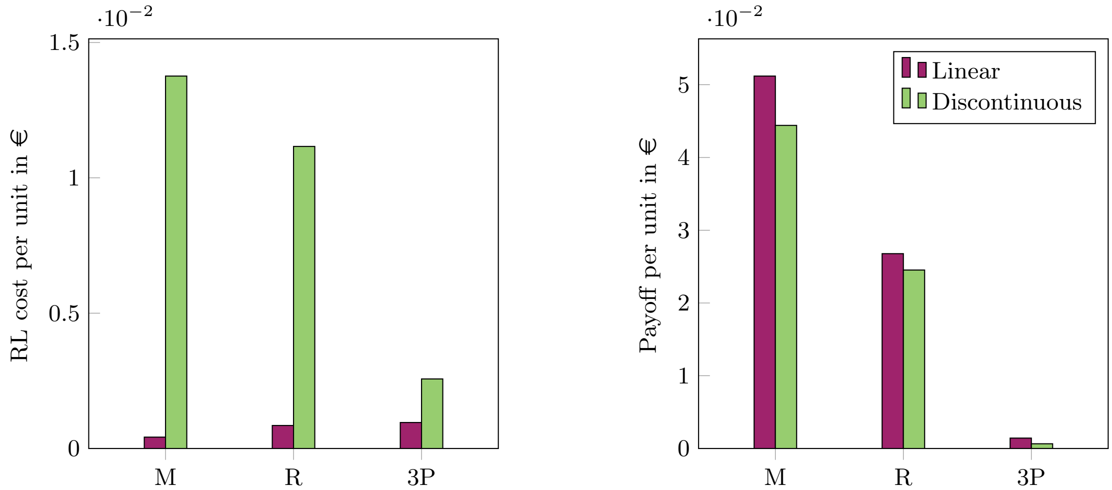
The attention will be drawn towards various scenarios, due to the influence of the external setting on an optimal solution. With all players incorporating the same high density, the manufacturer and the retailer are able to expand their profits through collecting more returns in the non-cooperative game. The third-party is not involved in the collection if all collecting agents have a customer density of 100%. Especially the manufacturer profits from an increase in customer density in the cooperative version of the game. However, if the customer density decreases for all players to a very low level, the collection of the third-party is not economic any more. Additionally, the payoffs of the manufacturer and the retailer decreases significantly, while cooperation almost become inevitable to achieve the highest payoffs obtainable in this scenario. The market size has a significant impact on the profit of the collecting agents. In the non-cooperative game a shrinking market size lowers profits for all three players. However, the decrease in payoff of the third-party is far less drastic than for the manufacturer and the retailer. Similar observations can be made for an increase in market size. While the payoff of each player rises rapidly, the third-party is able to increase its payoff the most while the manufacturer doubles and the retailer triples their profits. The collection rates change accordingly. Compared to the cooperative games, the manufacturer, the retailer and the third-party obtain the highest payoffs while collecting in the grand coalition. Therefore, the grand coalition is still most likely to form. Nevertheless, the two-player cooperations almost lead to similar results for a large market area. However, if the market area shrinks, especially the retailer and the third-party have to cooperate in some form to keep the collection profitable.

In conclusion, the stability of the non-cooperative and the cooperative version of the game shows similar results. In most cases of the cooperative game, the grand coalition is an optimal solution to keep profits high even with changing scenarios. Therefore, it is important to implement the idea of cooperation between different collecting options when designing a reverse logistics structure.

7.3.2 Current scientific knowledge versus thesis conclusions

The cost of collection is similar to all three collecting agents in the papers reviewed in Chapter 2 that serve as a framework for this thesis. Even though the question of the importance of transport cost is raised, none of the articles investigate further on the influence of reverse logistics cost individual to each collecting agent on the choice of channel structure. However, most other assumptions like the monopoly of the retailer, the non-existence of economies and diseconomies of scale, the perfect substitution of products as well as the long-life cycle of products are lifted in the development of research on this topic. That is why, the highly influential assumption on the similarity of collection cost was introduced and evaluated in the course of this thesis.

Comparing the reverse logistics cost and the payoffs for the linear logistics cost applied by Savaskan et al. [73] and the discontinuous and non-linear logistics cost established in this thesis lead to different insights. In the benchmark scenario, the reverse logistics cost in Figure 7.2(a) is lower for the linear model. The reverse logistics cost in the discontinuous model is higher for every player, due to the introduction of different transport options resulting in economies of scale and the influence of the cost necessary for operating the facilities. However, the manufacturer's reverse logistics cost is now the highest followed by the retailer and the third-party incorporating the lowest cost. This leaves a more realistic picture on the reverse logistics cost, since in the model of Savaskan et al. [73] the third-party, even though flexible in equipment and location, experienced the highest cost per unit. On the other hand, the change in payoff function, illustrated in Figure 7.2(b) resembles the conclusions from Savaskan et al. [73] with the manufacturer gaining the highest payoff followed by the retailer and the third-party still earning the lowest profit per unit.



(a) *Differences in RL cost per player* (b) *Differences in payoff per player*

FIGURE 7.2: *The differences in RL cost and payoff per unit between the model of Savaskan et al. [73] and the model developed in this thesis.*

Especially with the change of perspective to the manufacturer's point of view in Chapter 6, the results of the thesis extend the results of the papers. However, the general conclusion that closeness to the market is crucial to choose an optimal reverse channel structure stays consistent. In various scenarios, not only the reverse logistics cost and payoff per unit, but also per area in the form of cooperation amongst the manufacturer with either the retailer or the third-party is investigated. In the benchmark scenario, the best choice is the third-party. Due to the cheapest reverse logistics cost and the high customer density, the favour tips from the retailer as identified in the papers, to the third-party. Even with small changes in single parameters, this conclusion is valid. As for example with changes in fuel price increasing transport cost or rising fixed cost due to high land prices, the third-party is the most attractive choice amongst all collecting agents. Nevertheless, the retailer is the best choice in scenarios were the customer density is the same amongst all players, either very high or very low. However, for a changing market size, the third-party would be the collection partner of choice for the manufacturer. With the help of these investigations, the thesis shifts perception towards the importance of the reverse logistics cost in an optimal choice of the reverse channel structure.

7.3.3 An optimal reverse channel structure for the benchmark

Even though only the scenario prescribed by Savaskan et al. [73] and Fleischmann et al. [30] will be used to evaluate this question, the three perspectives of the best choice in the non-cooperative game, the cooperative game and from the manufacturer's point of view will be addressed. The focus lies on the two-fold motivation of economic profitability and compatibility with legislative regulations to sum up the observations.

In the non-cooperative version of the benchmark scenario, every collecting agent has the opportunity to use the reverse logistics structure that belongs to the player. Therefore, each player can only decide on the level of the collection rate. Looking at the payoff per unit, the manufacturer and the retailer would only engage with a low rate in the collecting efforts. The third-party would collect slightly more returns. Comparing the payoff per unit, the manufacturer would be able to achieve the highest profit, followed by the retailer and the third-party. The profit the manufacturer and the retailer are making from selling new products explains this observation. However, the third-party is able to achieve a profit, even without the sales of new products. With a shift in focus from the payoff per unit to the payoff per area, the collection rates increase for the manufacturer and the retailer in the benchmark scenario. The profit from selling new products allows a larger number of returns being profitable in this case. That is also why the collection rate of the third-party does not change. The manufacturer has the highest profit compared to the payoffs per area of the retailer who has a higher payoff than the third-party. However, the fact that the third-party can provide more than one client with the service of collecting returns and thus make profit from different channels should be included into the comparison. Therefore, it is shown that collecting returns can be a profitable business on its own.

In the cooperative version of the benchmark scenario, every collecting agent uses their own reverse logistics structure, but is able to cooperate with opponents that use their collecting options. The collecting agents need to return a certain percentage of the used products to comply with legislative regulations. Therefore, it would be the most profitable option for each collecting agent to cooperate collectively and form the grand coalition. Comparing the profit per area, the manufacturer would gain the highest payoff, followed by the retailer and the third-party. This resembles the conclusions drawn from the non-cooperative version of the game.

The perspective is shifted to the manufacturer's point of view as the manufacturer is most likely be held accountable by the legislative regulations for the collection of end-of-life products. For the manufacturer it would be the best choice to select the third-party to carry out the collection as the third-party has the highest customer density and thus is closest to the market. Nevertheless, using the retailer as a collecting agent can also have a profit increasing effect. That is why, the manufacturer should take a cooperation between the third-party and the retailer into consideration while designing an optimal reverse channel structure.

In general, the level of the return rate or the extend of cooperation decides on an optimal reverse channel structure. Therefore, the point of view determines the monetarily most profitable answer to the problem evaluated in this thesis.

7.4 An outlook on future studies

The purpose of this thesis is to show that reverse logistics add value to a company. Especially with the example of the third-party specialising on services for the collection of returns, the profitability of this business is emphasized. With an increase of interest in environmental issues and sustainable growth, legislative regulations about the collection of end-of-life products will be introduced all over the world. Therefore, a company should take the benefits of reverse logistics into consideration at an early stage of the product and supply chain design. Additionally, the different versions and scenarios show that it does make sense for a company to start discussions with different providers of collection services. The best solution can only be found by taking market specifics into consideration, due to the differences in the reverse logistics structures of the various providers and their ability to handle these challenges.

Future studies in this topic could apply the model developed in this thesis to a specific case to broaden the research. Thereby, the general conclusions could be confirmed with real-world data. Additionally, the possibilities to integrate forward and reverse logistics to a larger extend could be taken into consideration with the implementation of the model in an actual case. Especially the level of customer density for each player and the differences in customer density between the three players could be supported by actual data. The customer density is one of the most important parameters in choosing a reverse channel and can tip the collection in favour of the player with the highest customer density. Furthermore, the legislative regulations that set a collection level and impose a fine for not achieving this level could be validated in an actual case. The collection level as well as the fine are the most important parameters to incentivise cooperation amongst the players.

The cost function could be further improved to deepen the research. In the case of the retailer and the third-party, the level of transfer price could be adapted dynamically. Additionally, the impact of the components of the cost of investment in the collection effort could be further analysed. The assumption of the manufacturer taking the responsibility for the collection could be shifted towards the customers. This could lead to changes in the transport cost as the customer would then carry out the collection effort.

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